

14.06: Section Handout

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April 22, 2005

Today we will cover Angeletos (2004). This is a short paper, published in the NBER Macroeconomics Annual 2003, that presents a reduced-form model suitable to analyze fiscal policy issues like tax-smoothing and corrective taxes. With this model we can also see how the type of assets the government has access to affects fiscal decisions.

1 The Economy

The model is ad-hoc in the sense that none of the equations are derived from microfoundations, but they do capture the main trade-offs and features of a standard model. Giving up on formality, in the sense of a very thorough derivation of the model, allows us to get clear results that are easier to interpret and convey the same basic intuition we would get from more complicated models.

1.1 Social Welfare

The social welfare function is given by

$$u = - \sum_{t=0}^{\infty} \beta^t E_t [(y_t - y_t^*)^2 + \omega \pi_t^2] \quad (1)$$

where y_t^* is the (random) efficient level of output (first-best level), y_t is the actual (endogenous) level of output, π_t is the actual inflation (endogenous too), and ω reflects the welfare loss associated with the distortion in the cross sectoral allocation of resources because of the inflation level; and in fact the more inflexible prices are, the higher ω is.

1.2 Market Equilibrium

Equilibrium in the output markets is given by

$$y_t = -\psi \tau_t + \chi (\pi_t - \beta E_t \pi_{t+1}) + \varepsilon_t, \quad \psi, \chi > 0 \quad (2)$$

where the first term captures the distortionary effects of taxes (in terms of the final good), and the second term is just the effect of monetary policy when prices are sticky; in particular, the less flexible prices are, the higher χ is.¹ The last term, ε_t , is a cost-push shock.

1.3 The Government Budget

Under the assumption that the government can trade real and nominal bonds, freely determining the level of real bonds but keeping nominal bonds as a constant fraction of GDP (\bar{d} , which is considered a parameter of the model) we can write the government budget as²

$$b_{t-1} = \tau_t + z_t + \underbrace{\bar{d}(\pi_t - \beta E_t \pi_{t+1})}_{\text{gains from unexpected } \pi} - g_t + \beta b_t \quad (3a)$$

where b_t is the total level of debt as fraction of GDP, τ_t is the tax rate on aggregate income. The term z_t will make the difference in the three cases we will analyze. It represents any state-contingent lump-sum transfers the government may have access to. The cases are

1. unrestricted lump-sum taxation: the government can freely choose any level of z_t it wants;
2. no lump-sum taxation but complete insurance: in this case the only restriction is that $E_{t-1} z_t = 0$;
3. no lump-sum taxation and no insurance: $z_t = 0$, no matter what happens.

2 Optimal Policy

2.1 The Ramsey Problem

We can now formulate the Ramsey problem

$$\begin{aligned} \min \quad & E_0 \sum_{t=0}^{\infty} \beta^t [(y_t - y_t^*)^2 + \omega \pi_t^2] \\ \text{s.t.} \quad & y_t = -\psi \tau_t + \chi (\pi_t - \beta E_t \pi_{t+1}) + \varepsilon_t \\ & b_{t-1} - \beta b_t = \tau_t + z_t - g_t + \bar{d} (\pi_t - \beta E_t \pi_{t+1}). \end{aligned}$$

¹Notice that (2) can be rewritten as

$$\pi_t = \beta E_t \pi_{t+1} + \frac{1}{\chi} (y_t - y_t^n)$$

where $y_t^n \equiv -\psi \tau_t + \varepsilon_t$.

²See footnote 5 in the paper.

The Lagrangean of the problem is given by $\mathcal{L} = \sum_{t=0}^{\infty} \beta^t E_t [L_t]$ where L_t is given by

$$L_t \equiv \frac{1}{2} [(y_t - y_t^*)^2 + \omega \pi_t^2] - \mu_t [\chi (\pi_t - \beta \pi_{t+1}) - (y_t + \psi \tau_t + \varepsilon_t)] + \lambda_t [(b_{t-1} - \beta b_t) - (\tau_t - g_t + z_t) - \bar{d} (\pi_t - \beta \pi_{t+1})]. \quad (4)$$

Taking FOC with respect to y_t , π_t , b_t , and τ_t , and doing some algebra we obtain the following expressions

$$y_t - y_t^* = -\frac{1}{\psi} \lambda_t \quad (5)$$

$$\pi_t = \frac{1}{\omega} \left(\frac{\chi}{\psi} + \bar{d} \right) (\lambda_t - \lambda_{t-1}) \quad (6)$$

$$\lambda_t = E_t \lambda_{t+1}. \quad (7)$$

Equations (2), (3a), (5), (6), and (7) plus the initial condition $b_{-1} = \bar{b}$ pin down the optimal plan. From (6) and (7) is clear that $E_t \pi_{t+1} = 0$. The key variable in the solutions is λ_t , the shadow cost of government budget, because it will determine the value of the output gap and optimal inflation rate. Furthermore, λ_t follows a random walk, reflecting the intertemporal smoothing done with the riskless bonds the government can trade.

2.2 A Benchmark: The First Best

Take also the FOC with respect to z_t

$$\begin{aligned} \beta^t E_t \lambda_t &= 0 \\ \Rightarrow \lambda_t &= 0 \quad \forall t. \end{aligned} \quad (8)$$

Using equation (8), and equations (5) to (7), we can obtain

$$y_t - y_t^* = 0 \quad (9)$$

$$\pi_t = 0. \quad (10)$$

Take the aggregate supply

$$y_t = -\psi \tau_t + \chi (\pi_t - \beta E_t \pi_{t+1}) + \varepsilon_t,$$

and plug in equations (9) and (10), to obtain

$$y_t^* = -\psi \tau_t + \chi (\pi_t - \beta E_t \pi_{t+1}) + \varepsilon_t$$

$$y_t^* = -\psi \tau_t + \varepsilon_t$$

where I have imposed that $\pi_t = 0 \forall t$, so $E_t \pi_{t+1} = 0$ too. And rearranging terms I get

$$\tau_t^* \equiv -\frac{1}{\psi} (y_t^* - \varepsilon_t). \quad (11)$$

It is intuitive. A shock to y_t^* , makes the government wishing to be able to increase demand, so it would reduce distortionary taxation τ_t to push output up to the first-best level of output. The fiscal impact is partly compensated with the lump-sum taxes. On the other hand, a shock to ε_t will induce a higher output level y_t , for a certain level of τ_t and π_t ; given that y_t^* is still the same, then the government would like to offset this shock to aggregate supply and will try to keep $y_t^n = 0$. Depending on the sign and magnitude of the shocks the government may set a tax or a subsidy, which aims to correct distortion in the supply side, that's the reason why we can call this tax rate τ_t^* a *Pigouvian tax*.

The policymaker uses the lump-sum taxes to balance the budget: $z_t = (g_t - \tau_t^*) + (1 - \beta)\bar{b}$.

2.3 Optimal Policy with Complete Markets

In this case the government does not have access to lump-sum taxes, but it can issue state-contingent debt, thus it can replicate full insurance against shocks. In the model this is equivalent to the government choosing z_t with the restriction that $E_{t-1}z_t = 0$. We can set up the Lagrangean and take the FOCs. The FOC with respect to z_t and equation (7) imply that

$$\lambda_t = \bar{\lambda}, \quad \forall t \text{ and states.} \quad (12)$$

Equation (12) has a clear intuition. Without access to lump-sum taxes the government is left only with distortionary tools, so any action necessarily implies some distortion in the economy. But with access to a complete market of contingent claims the government can isolate from random shocks and so will choose to smooth away any problem associated to the random shocks, in this case this means choosing a path of endogenous variables such that the shadow cost of fiscal balance is constant across time *and* states.

Using (12) we can obtain the following expressions

$$y_t - y_t^* = -\frac{1}{\psi}\bar{\lambda} < 0 \quad (13)$$

$$\pi_t = 0 \quad (14)$$

$$\tau_t = \frac{1}{\psi^2}\bar{\lambda} + \tau_t^*, \quad (15)$$

where τ_t^* is the Pigou tax given by equation (11).

$$\begin{aligned} z_t &= (g_t - \tau_t) - (\bar{g} - \bar{\tau}) \\ &= (g_t - \bar{g}) - (\tau_t - \bar{\tau}), \end{aligned} \quad (16)$$

where $\bar{g} \equiv E g_t$ and $\bar{\tau} \equiv E \tau_t$. So, as we have expected, z_t will absorb any variation in the conditions of the economy, which exactly what we should expect of a variable that reflects insurance against shocks. Thus, with z_t caring about the fluctuations

and shocks, the shadow cost of the government budget will be isolated from any shock that hits the budget constraint, and will be given by

$$\bar{\lambda} = \psi^2 [(1 - \beta) \bar{b} + (\bar{g} - \bar{\tau}^*)], \quad (17)$$

which implies that

$$y_t - y_t^* = -\psi [(1 - \beta) \bar{b} + (\bar{g} - \bar{\tau}^*)].$$

An additional result can be obtained using (15) and (17),

$$\tau_t = \tau + (\bar{\tau}_t^* - \bar{\tau}^*) \quad (18)$$

where $\tau = \bar{g} + (1 - \beta) \bar{b}$ is exactly the optimal tax rate in a neoclassical economy without any distortion like the ones introduced in this model. In particular, the second term in equation (18) reflects the effect of the Keynesian business cycle component, the government wants to correct the inefficient component of the business cycles, allowing output to variate if and only if y_t^* moves.