14.06: Section Handout

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1 The Model

1.1 The Setup

Consider a set-up like in Barro and Gordon (1983)

$$y_t - y = \theta \left(\pi_t - \pi_t^e\right), \qquad \theta > 0 \tag{1}$$

Equation (1) represents the supply side economy (a Phillips curve type of equation), where y_t is the actual level of output, y is the natural level of output, π_t is the inflation rate, and π_t^e is the expected inflation rate. The natural rate of output in this model is the output level we would observe is there were no surprises in inflation.

$$L = \frac{1}{2}\left(\alpha \pi_t^2 + (y_t - \gamma y)^2\right), \qquad \alpha > 0,\ \gamma > 1. \tag{2}$$

Equation (2) is the policymaker's loss function which is assumed to represent the society's preferences over output and inflation; α represents the relative weight society assigns to inflation deviations.[1] The term $(y_t - \gamma y)$ represents the deviations of actual output with respect to a "bliss" level which is assumed to be higher than the natural level, probably reflecting some friction in the markets.

1.2 Timing

We will consider a situation where all players know y and the set-up of the economy, given by equations (1) and (2).

This is a one period model, agents choose π_t^e, and then the policymaker will chooses the π_t that maximizes (2) subject to (1) and π_t^e.

1.3 Discretionary Policy Equilibrium

The agents and the policymaker play a sequential move game, so we can find the equilibrium outcome using backwards induction. Consider first the policymaker's

[1] Notice that we have assumed that the "desired" level of inflation $\pi^* = 0$. This assumption is not crucial and will simplify the algebra.

problem

$$\min_{\pi_t} \quad \frac{1}{2}\alpha\pi_t^2 + \frac{1}{2}\left(\theta\left(\pi_t - \pi_t^e\right) - y\left(\gamma - 1\right)\right)^2 \tag{P1}$$

The FOC is

$$\alpha\pi_t + \theta\left[\theta\left(\pi_t - \pi_t^e\right) - y\left(\gamma - 1\right)\right] = 0.$$

The optimal inflation rate, as a function of π_t^e is

$$\pi_t\left(\pi_t^e\right) = \left(\frac{\theta^2}{\theta^2 + \alpha}\right)\pi_t^e + \left(\frac{\theta}{\theta^2 + \alpha}\right)\left(\gamma - 1\right)y > 0. \tag{3}$$

Equation (3) corresponds to the policymaker's best response function.[2]

We can now analyze the agents' problem. In this game with complete and perfect information, the agents' take the policymaker's best response function into account when setting π_t^e. Assume that the agents' payoff is given by

$$U = -\frac{1}{2}\left(\pi_t - \pi_t^e\right)^2, \tag{4}$$

then, the agents' optimization problem is

$$\max_{\pi_t^e} \quad -\frac{1}{2}\left(\left(\frac{\theta^2}{\theta^2 + \alpha}\right)\pi_t^e + \left(\frac{\theta}{\theta^2 + \alpha}\right)\left(\gamma - 1\right)y - \pi_t^e\right)^2. \tag{P2}$$

Clearly, the expected inflation rate that maximizes this function is the one that makes the expression in the parenthesis equal to 0. This rate is given by

$$\widehat{\pi}_t^e = \frac{\theta}{\alpha}\left(\gamma - 1\right)y. \tag{5}$$

In equilibrium, agents' will set a positive expected inflation rate. The optimal expected inflation rate is such that $\pi_t\left(\widehat{\pi}_t^e\right) = \widehat{\pi}_t^e$. In fact this result is exactly what we would obtain if we assume that agents are rational; under this assumption, equilibrium we must have $\pi_t = \pi_t^e$,[3] thus we can find π_t^e as the fixed point of equation (3).

As a result, the policymaker is totally unable to increase the output level of the economy on a permanent base, and her inability to commit to a low inflation rate leads to an equilibrium where output equals its natural rate ($y_t = y$), but the inflation rate is inefficiently high. Agents know the government cannot commit and also know the government has an expansionary (inflationary) bias, they anticipate that setting a higher expected inflation rate $\widehat{\pi}_t^e$.

[2] The slope of the best response function is bounded between 0 and 1. The intercept is strictly positive.

[3] In general we would require $E\left(\pi_t\right) = \pi_t^e$, given that there are no stochastic shocks in this model, then $E\left(\pi_t\right) = \pi_t$.

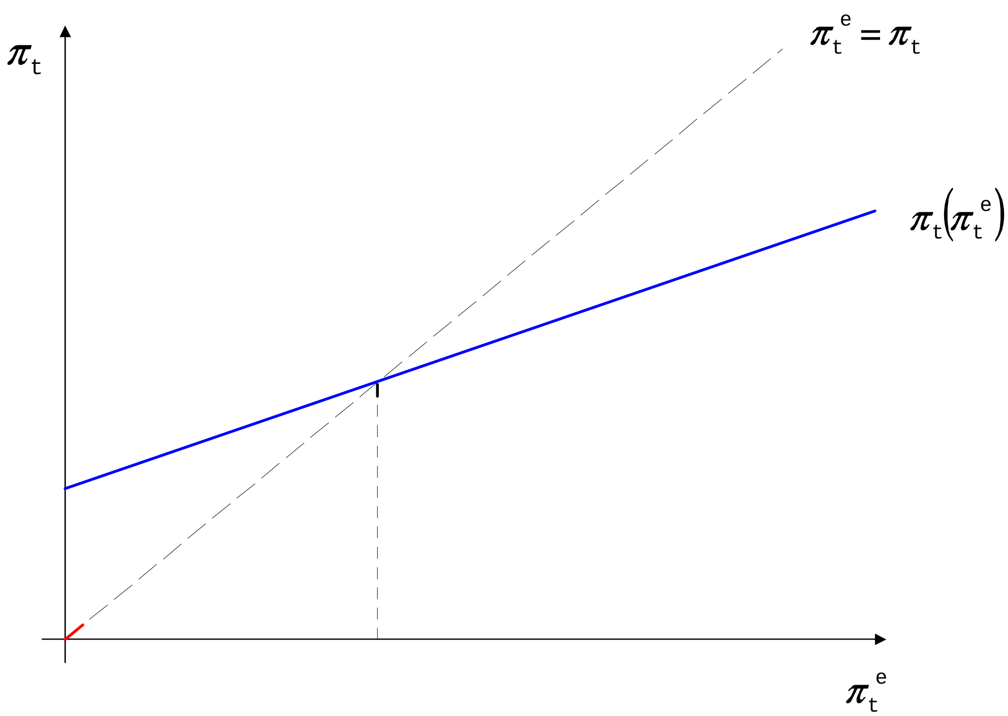

Figure 1: Equilibrium in the Deterministic Game.

Finally, social loss in the discretionary equilibrium is

$$\begin{aligned} L^D &= \frac{1}{2}\alpha\left(\frac{\theta}{\alpha}(\gamma-1)y\right)^2 + \frac{1}{2}(1-\gamma)^2 y^2 \\ &= \frac{1}{2}\left[\left(\frac{\theta^2}{\alpha}+1\right)(\gamma-1)^2 y^2\right] \\ L^D &= \left(\frac{1}{2}\right)\frac{(\gamma-1)^2 y^2}{\lambda}, \end{aligned} \tag{6}$$

where $\lambda \equiv \frac{\alpha}{\alpha+\theta^2} < 1$. The social loss is increasing in the discrepancy between the natural and the desired ("socially optimal") output level, and in the slope of the aggregate supply (Phillips curve), because this implies a higher temptation to generate surprises in inflation. It is decreasing in α, reflecting that inflation is relatively more costly for the policymaker, thus less incentives to surprise with unexpected inflation.

Why is people anticipating the "inflationary bias"? You can fool some people some of the time, but you cannot fool all of them, all the time. They will learn and adjust to it.

1.4 Equilibrium with Commitment

Imagine that the government has access to a technology that makes announcements totally credible, and allow the government to have a first move where it announces the inflation rate that will be observed in the economy.[4] If the announcement is credible, the best the government can do is to announce that will set $\pi_t = 0$; agents will then set $\pi_t^e = 0$ and the government will effectively adjust policies to make $\pi_t = 0$. So, the outcome in this case would be $y_t = y$ and $\pi_t = 0$.

The social loss is given by

$$L^R = \frac{1}{2}\left(\gamma - 1\right)^2 y^2, \tag{7}$$

which is smaller than L^D, because $\lambda < 1$. We can conclude then that the society would be better off if the policymaker were able to commit not to inflate.[5]

2 Stochastic Model

We can interpret the commitment solution of section 1.4 as the solution when the government can self-impose a strict inflation rule. Our model also tells us, that a solution like that is preferable to a solution where the government can actually use monetary policy. If this is true, why do we observe countries using discretionary policy (ex. USA).

We can modify our model allowing nature to play a role in this game. Modify the timing as follows: after the agents have set expected inflation, the nature moves with a random shock z_t which is publicly observed at zero cost. The policymaker moves after the shock is realized, and will take into account the shock when setting π_t.

Now, the aggregate supply is given by

$$y_t - y = \theta\left(\pi_t - \pi_t^e\right) + z_t, \qquad \theta > 0,\ E\left(z_t\right) = E_{t-1}\left(z_t\right) = 0,\ V\left(z_t\right) = \sigma_z^2. \tag{8}$$

The solution looks as follows:

$$\pi_t\left(\pi_t^e, z_t\right) = \left(1 - \lambda\right)\left[\pi_t^e - z_t/\theta + \left(\gamma - 1\right) y/\theta\right] \tag{9}$$

$$\pi_t^e = \left(\frac{1 - \lambda}{\lambda}\right)\frac{\left(\gamma - 1\right) y}{\theta} \tag{10}$$

$$\pi_t = \left(\frac{1 - \lambda}{\lambda}\right)\frac{\left(\gamma - 1\right) y}{\theta} - \frac{\left(1 - \lambda\right)}{\theta} z_t = \pi_t^e - \frac{\left(1 - \lambda\right)}{\theta} z_t. \tag{11}$$

[4] An alternative motivation would be to allow modify the game making the government move first, and the agents move second; as you can see, "expected" is not exactly the best description for the agents' action.

[5] The discussion on rules and discretionary policy was emphasized by Kydland and Prescott (1977) in a more general context, monetary policy being one of the possible cases where this issue arises.

Notice that

$$\theta\left(\pi_t - \pi_t^e\right) = -\left(1-\lambda\right) z_t,$$

and

$$y_t = y + \lambda z_t. \tag{12}$$

Equation (12) shows that in this case discretionary policy allows the policymaker to cushion the supply shock z_t, reducing its effect on output by setting a higher inflation rate, π_t.

Consequently, for a given z_t, the social loss is

$$L = \frac{1}{2}\left(\frac{1-\lambda}{\lambda}\right)\left[\left(\gamma-1\right)y - \lambda z_t\right]^2 + \frac{1}{2}\left[y\left(1-\gamma\right)+\lambda z_t\right]^2,$$

taking expected values

$$EL^D = \left(\frac{1}{2}\right)\left[\frac{\left(\gamma-1\right)^2 y^2}{\lambda} + \lambda\sigma^2\right]. \tag{13}$$

If the policymaker commits to $\pi_t = 0$, the strict inflation rule, the equilibrium is

$$\begin{aligned} \pi_t &= \pi_t^e = 0 \\ y_t &= y + z_t \qquad (14) \\ L &= \frac{1}{2}\left[y\left(1-\gamma\right)+z_t\right]^2. \qquad (15) \end{aligned}$$

The expected social loss is

$$EL^R = \frac{1}{2}\left(y^2\left(\gamma-1\right)^2+\sigma^2\right). \tag{16}$$

2.1 Which regime is better?

We can now compare both regimes. If we take the unconditional expectation of the loss as the welfare criterion, then we just need to compare equations (13) and (16).

$$\begin{aligned} EL^R &< EL^D \\ \frac{1}{2}\left[y^2\left(\gamma-1\right)^2+\sigma^2\right] &< \left(\frac{1}{2}\right)\left[\frac{\left(\gamma-1\right)^2 y^2}{\lambda}+\lambda\sigma^2\right] \\ y^2\left(\gamma-1\right)^2+\sigma^2 &< \frac{\left(\gamma-1\right)^2 y^2}{\lambda}+\lambda\sigma^2 \\ \lambda\sigma^2 &< \left(\gamma-1\right)^2 y^2 \\ \left(\gamma-1\right)y &> \sigma\sqrt{\lambda}. \qquad (17) \end{aligned}$$

Now, it is not true that the strict rule would always be preferable to the discretionary solution. In fact, this just reflects a trade-off, inflationary bias versus the ability to cushion the shocks. Equation (17) just tells us the exact criterion for this.

3 Extensions

1. Inflation Targeting;
2. Repeated Games and Reputation.

References

Barro, R., and D. Gordon (1983); "A Positive Theory of Monetary Policy in a Natural Rate Model", *Journal of Political Economy*, 91(4), 589-610.

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