ANGELETOS AND WERNING (2005)

"CRISES AND PRICES"

(1) MODEL SET UP

Agents: $i \in [0,1]$ continuum of agents

two actions: $a_i = 1 \rightarrow$ attack

$a_i = 0 \rightarrow$ not attack

Current situation: "status-quo"

payoff from not attacking is 0

payoff from attacking is $\begin{cases} 1-c > 0 & \text{if "status-quo" is abandoned} \\ -c & \text{if not} \end{cases}$

$c \in (0,1)$ is the cost of attacking

"Status-quo" is abandoned if $A > \theta$, where A is the mass of agents attacking, and θ is an exogenous fundamental that represents the strength of the status quo.

Define $U(a_i, A, \theta) = a_i \cdot (R(A,\theta) - c)$ as the payoff to agent i,

where $R(A,\theta)$ is the regime outcome $\begin{cases} 1 & \text{if } A > \theta \\ 0 & \text{otherwise} \end{cases}$

$U(1, A, \theta) - U(0, A, \theta)$ increases with A

$\Rightarrow$ coordination motive due to complementarity: payoff of attacking increases with the mass of attackers.

(A) Benchmark Case: Public Information About θ

In the case when θ is public information, $A=1$ and $A=0$ are an equilibrium whenever $\theta \in [\underline{\theta}, \bar{\theta}] \equiv (0,1]$

(B) Imperfect observability

Timing:

0 — signals — 1

Nature plays $\downarrow$ draws θ from a known distribution

Agents play and decide whether to attack

There will be 2 signals, private and common; both are imperfect signals about θ.

(B.1) Exogenous Information

$x_i = \theta + \sigma_x \xi_i$: private signal

$\sigma_x > 0$ $\quad \xi_i \sim N(0,1)$ independent of θ

$z = \theta + \sigma_z v$: common, exogenous public signal

$\sigma_z > 0$, $\quad v \sim N(0,1)$ independent of θ and ξ_i

Define $\left. \begin{array}{l} \alpha_x = \sigma_x^{-2} \\ \alpha_z = \sigma_z^{-2} \end{array} \right\}$ as the precisions of private and public info.

Equilibrium notion: monotone equilibrium, defined as perfect Bayesian equilibria such that, for a given z, an agent attacks iff the realization x of his private signal is less than some threshold $x^*(z)$.

<u>Proposition 1</u> (Morris-Shin)

In the game with exogenous info, the equilibrium is unique.

iff $0 \quad \sigma_x \leq \sigma_z^2 \sqrt{2\pi}$

Proof: agents attack iff $x_i < x^*(z)$

$\Rightarrow$ size of the attack is $A(\theta, z) = \Phi\left(\sqrt{\alpha_x}\,(x^*(z) - \theta)\right)$

where Φ is the cdf of a normal (standard) distribution.

The status-quo is abandoned iff $\theta \leq \theta^*(z)$, where $\theta^*(z)$ is the solution to $A(\theta, z) = \theta$ or

$$x^*(z) = \theta^*(z) + \frac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*(z))$$

$A(\theta, z) \longrightarrow$ all the guys that perceive the fundamental to be "sufficiently weak" ...

$A(\theta, z)$ is decreasing in θ

$\Rightarrow$ for any $\theta \leq \theta^*(z)$, $A(\theta, z) > \theta$ and the status quo is abandoned.

(1) $\quad x^*(z) = \theta^*(z) + \frac{1}{\sqrt{\alpha_x}} \Phi^{-1}(\theta^*(z)) \longrightarrow$ relation between x^* and θ^* such that the attack effectively leads to abandon the status quo if $\theta < \theta^*$

Now we need to be sure that the agents are indifferent between attacking and not attacking:

$$\Pr[\theta \leq \theta^*(z) \mid x, z] = c$$

Posteriors about θ are

$$N\left(\frac{\alpha_x}{\alpha_x + \alpha_z} x + \frac{\alpha_z}{\alpha_x + \alpha_z} z,\ \frac{\sigma_z^2 \sigma_x^2}{\sigma_x^2 + \sigma_z^2}\right)$$

with precision $\alpha = \alpha_x + \alpha_z$

(2) $$\Phi\left[\sqrt{\alpha_x+\alpha_z}\left(\theta^*(z)-\frac{\alpha_x}{\alpha_x+\alpha_z}X^*(z)-\frac{\alpha_z}{\alpha_x+\alpha_z}z\right)\right]=c$$

Equilibrium is determined by the joint solution of (1) and (2):

(3) $$-\frac{\alpha_z}{\sqrt{\alpha_x}}\theta^*+\Phi^{-1}(\theta^*)=g(z)=\sqrt{1+\frac{\alpha_z}{\alpha_x}}\,\Phi^{-1}(1-c)-\left(\alpha_z/\sqrt{\alpha_x}\right)z$$

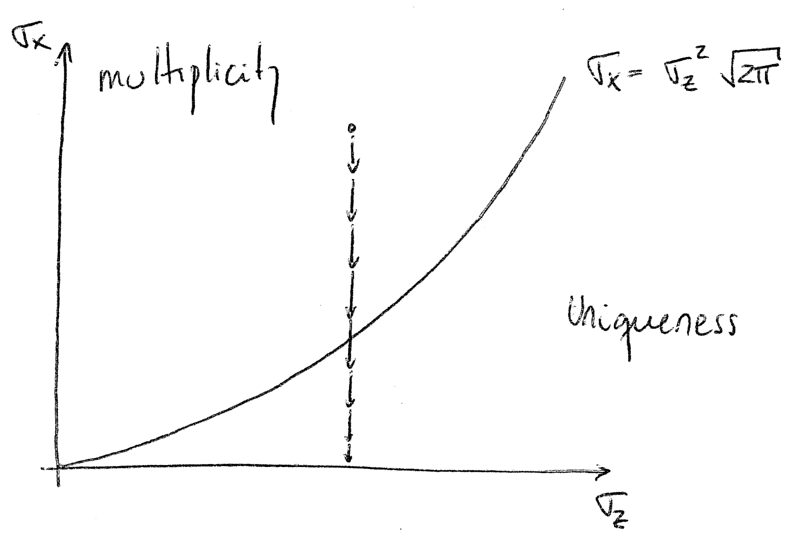
Equation (3) can always be solved and the solution is unique iff

$$\frac{\alpha_z}{\sqrt{\alpha_x}}\le\frac{1}{\sqrt{2\pi}}\iff\boxed{\sigma_x\le\sigma_z^2\sqrt{2\pi}}$$ //

Corollary 1

In the limit as $\sigma_x\to 0$, there is a unique equilibrium in which $R(\theta,z)\to 1$ if $\theta<\hat{\theta}$ and $R(\theta,z)\to 0$ if $\theta>\hat{\theta}$, where $\hat{\theta}=1-c$.

This corollary tells us that small perturbation away from perfect information ($\sigma_x=0$ is perfect information) suffice to obtain a unique equilibrium.



Note: along the x-axis we have multiplicity. (why?)

We can read the result as follows, for any $\sigma_z>0$ we can always find a σ_x small enough such that there is a unique equilibrium in the coordination game we described.

The intuition for this result is clear. Think in terms of the precision of the signals. Rewrite the condition in proposition 1

$$\alpha_z \leq \sqrt{\alpha_x} \cdot \eta \qquad \eta = \sqrt{2\pi}$$

So, there is a unique equilibrium if the private signal is "precise enough". An agent i observes x_i and z, and forms "posterior distribution" for θ, and the mean is given by $\frac{\alpha_x}{\alpha_x+\alpha_z} \cdot x_i + \frac{\alpha_z}{\alpha_x+\alpha_z} z$.

The higher the precision of her own signal the less influential the common info (z) is. Private signals "anchor" the posteriors and the agent is not thinking about what other people is doing, she is very confident about the possible values of θ and there isn't much uncertainty about what the other agents' actions will be. $\Rightarrow$ the basic mechanism behind multiple equilibria breaks down.

(B.2) Endogenous Information: Financial Prices

Exactly as before, nature draws θ and the agents receive a private signal x_i. However, we will not take z to be exogenous but it will be endogenously determined.

Assume that there is a risky asset which is traded at a price p; this asset pays a dividend f which we will assume to be exogenous and equal to θ.

Preferences: agents have CARA utility: $-\frac{e^{-\gamma c_i}}{\gamma}$, $\gamma > 0$

consumption $\longrightarrow$ $c_i = W - pk_i + fk_i$ $\qquad$ k_i: investment in the risky asset

CARA utility: constant absolute risk aversion.

Asset supply: $K^s(\varepsilon) = \sigma_\varepsilon \varepsilon$ — uncertain and not observable net supply of the asset.

$\sigma_\varepsilon > 0$

$\varepsilon \sim N(0,1)$ independent of θ and ξ_i

$\varepsilon \rightarrow$ introduces noise in the information revealed by the equilibrium prices.

σ_ε: parameter for the exogenous noise in the aggregation process.

Think of this new setup in the following way:

1^{st} Stage: agents trade assets and observe the market equilibrium

2^{nd} Stage: agents play the same we just described, choose whether to attack.

Asset demands and attack decisions are functions of x and p, but the aggregate variables are functions of θ and p.

The formal definition in the paper:

Definition 1:

An equilibrium is a price function, $P(\theta,\varepsilon)$, individual strategies for investment and attacking, $k(x,p)$ and $a(x,p)$, and their corresponding aggregates, $K(\theta,p)$ and $A(\theta,p)$, such that:

rational expectations competitive eqm. for 1^{st} stage

$$\begin{cases} k(x,p) \in \underset{k\in\mathbb{R}}{\arg\max}\ \mathbb{E}\left[V((f-p)k) \mid x,p\right] & \text{extracts info from prices} \\ K(\theta,p) = \int_x k(x,p)\,\phi\left(\frac{x-\theta}{\sigma_x}\right) dx & \\ K(\theta, P(\theta,\varepsilon)) = K^s(\varepsilon) & \longrightarrow \text{mkt. clearing} \end{cases}$$

Perfect Bayesian eqm. for 2^{nd} stage.

$$\begin{cases} a(x,p) \in \underset{a\in[0,1]}{\arg\max}\ \mathbb{E}\left[U(a, A(\theta,p), \theta) \mid x,p\right] \\ A(\theta,p) = \int_x a(x,p)\,\phi\left(\frac{x-\theta}{\sigma_x}\right) dx \end{cases}$$

And Let $R(\theta,\varepsilon) = R(\theta, A(\theta, P(\theta,\varepsilon)))$ denote the equilibrium regime outcome.

We want to focus in the particular case where $f=\theta$.

We will assume a linear price function.

The equilibrium price will be the same as a signal with precision α_p, that is observed by all the agents.

The posterior distributions of θ conditional on x and p are Normal with mean $\delta x + (1-\delta)p$ and precision $\alpha = \sigma_p^{-2}$, where $\delta = \frac{\alpha_x}{\alpha}$ and $\alpha = \alpha_x + \alpha_p$.

Asset demands are

$$k(x,p) = \frac{E[f \mid x,p] - p}{\gamma \, Var[f \mid x,p]} = \frac{\delta \alpha}{\gamma} \cdot (x-p)$$

Notice that all agents take the price as given.

Thus, aggregate demand is

$$K(\theta,p) = \frac{\delta\alpha}{\gamma}(\theta - p)$$

And imposing market clearing condition

$$K(\theta,p) = K^s(\varepsilon) = \sigma_\varepsilon \cdot \varepsilon$$

$$\Rightarrow P(\theta,\varepsilon) = \theta - \sigma_p \varepsilon$$

$$\sigma_p = \left(\frac{\delta\alpha}{\gamma}\right)^{-1} \sigma_\varepsilon$$

So, conditional on θ, p is normally distributed.

Using the definitions for α and δ we can compute a formula connecting σ_p and σ_x :

$$\delta = \frac{\alpha_x}{\alpha} \quad \Rightarrow \quad \sigma_p = \left(\frac{\partial \alpha}{\gamma}\right)^{-1} \sigma_\varepsilon$$

$$= \left(\frac{\alpha_x}{\gamma}\right)^{-1} \sigma_\varepsilon$$

$$= \frac{\gamma}{\alpha_x} \sigma_\varepsilon$$

$$\Rightarrow \boxed{\sigma_p = \gamma \sigma_\varepsilon \sigma_x^2} \qquad (*)$$

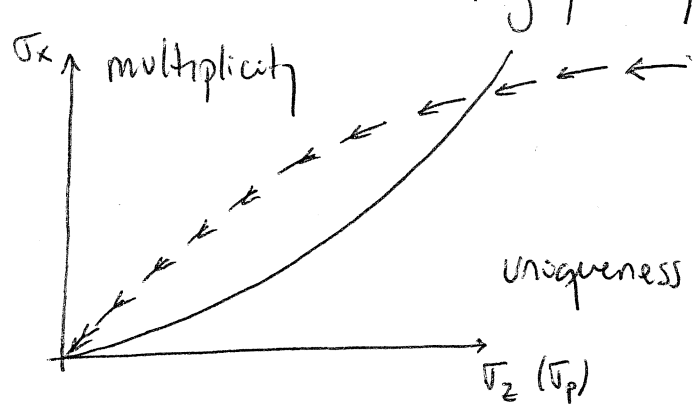
In this new setup, σ_p plays the same σ_z played before, but σ_p is not independent of σ_x! So our previous analysis cannot take σ_p as given.

Proposition 2

In the asset economy with exogenous dividend $f=\theta$, there are multiple equilibria if $\sigma_\varepsilon^2 \sigma_x^3 < \gamma^{-2}(2\pi)^{-1/2}$.

To obtain this result, just plug $(*)$ in the condition stated in Proposition 1.

We can show the result graphically:



Solid line:
$\sigma_x = \sigma_z^2 \sqrt{2\pi}$

Dashed line:
$\sigma_p = \gamma \sigma_\varepsilon \sigma_x^2$

Our previous analysis about reducing σ_x doesn't apply here; now we have that improving private signals (better info) also improves the public information, and it does it in a way such that multiplicity is ensured for σ_x close enough to 0.