Chapter 8

Financial Markets, Savings, and Growth

8.1 A Simple AK Model with Incomplete Markets

8.1.1 Model Setup

- A simple AK endogenous-growth model with incomplete markets and uninsured idiosyncratic risk, with or without aggregate uncertainty.

- A continuum of entrepreneurs/agents, $i \in [0, 1]$.

- Each period t, entrepreneur i has access to two technologies:

  - A common 'subsistence' or 'storage' technology, which is riskless,

$$G(k) = Rk \ , \quad R > 0$$

– (ii) An AK-type technology with individual- or project-specific risk,

$$f_t^i(k) = A_t^i k$$

where A_t^i is an idiosyncratic productivity shock. c.d.f. F and support $\mathbb{A} = \{A \in \mathbb{R} | F(A) \geq 0\} \subseteq \mathbb{R}_+$, $F(0) = 0$.

- A_t^i is i.i.d. across i and t, with c.d.f. F over $\mathbb{R}_+$,

$$\overline{A} \equiv EA_{t+1}^i = E_t[A_{t+1}^i] \ , \qquad \overline{A} > R > \inf\{A : F(A) > 0\}.$$

W.l.o.g., $R \geq 1/\beta > 1$.

- Parametrize distribution of A_t^i by σ:

$$A_t^i = \overline{A} \exp\left\{\sigma \varepsilon_t^i\right\}$$

ε_t^i is log-normal.

- Infinite horizon, Epstein-Zin preferences:

$$u_t = U(c_t) + \beta \cdot UV^{-1}\left(\mathbb{E}_t\left[VU^{-1}(u_{t+1})\right] \ \right)$$

- CEIS/CRRA preferences:

$$\begin{aligned} U(c) &= \frac{c^{1-1/\theta} - 1}{1 - 1/\theta} \\ V(c) &= \frac{c^{1-\gamma} - 1}{1 - \gamma} \end{aligned}$$

γ coefficient of relative risk aversion; θ elasticity of intertemporal substitution.

- The aggregates:

$$C_t \equiv \int_i c_t^i, \quad K_t \equiv \int_i k_t^i, \quad Y_t \equiv \int_i y_t^i \equiv C_t + K_t$$
$$g = \frac{C_{t+1}}{C_t} = \frac{K_{t+1}}{K_t} = \frac{Y_{t+1}}{Y_t}.$$

8.1.2 Optimal Individual Behavior

- To simplify, the entrepreneur has to fully specialize in one technology: a discrete employment (or portfolio) choice $l_t \in \{0, 1\}$.

- The budget constraints:

$$c_t^i + k_t^i \ \leq \ y_t \,, \qquad l_t \in \{0, 1\},$$

where

$$y_t = l_{t-1}^i A_t^i k_{t-1}^i + (1 - l_{t-1}^i) R k_{t-1}^i = \left[R + l_{t-1}(A_t^i - R) \right] k_{t-1}^i$$

- Optimal specialization l_t^i:

$$l_t^i = \arg \max_{l_t^i \in \{0,1\}} V^{-1} \left(E_t[V(c_{t+1}^i)] \right)$$

- Solution independent of i and t as long as A_{t+1}^i is i.i.d. across i and t :

$$\begin{aligned} l_t^i &= l \\ c_t^i &= (1-s) y_t^i \\ k_t^i &= s y_t^i = s \left[R + l(A_t^i - R) \right] k_{t-1}^i \end{aligned}$$

for some constant $l \in \{0, 1\}$ and $s \in (0, 1)$.

- Optimal s and l such that

$$s = \beta^{\theta} \left(\left\{ E_t[R + l(A_{t+1} - R)]^{1-\gamma} \right\}^{\frac{1}{1-\gamma}} \right)^{\theta - 1}$$

$$l = \arg \max_{l \in \{0,1\}} \left\{ l \cdot \left[E_t[A_{t+1}^{1-\gamma}] \right]^{\frac{1}{1-\gamma}} + (1 - l') \cdot R \right\}$$

- Define B as the certainty equivalent of the return to the risky technology (the risk-adjusted return):

$$B \equiv \left[E[A^{1-\gamma}] \right]^{\frac{1}{1-\gamma}} = \left[E_t[(A_t^i)^{1-\gamma}] \right]^{\frac{1}{1-\gamma}} \equiv B(\sigma)$$

- Note that B decreases with σ,

$$\frac{\partial B(\sigma)}{\partial \sigma} < 0$$

and satisfies

$$B(0) = \overline{A} > R > 0 = B(\infty)$$

Thus there is a unique $\widetilde{\sigma} \in (0, \infty)$ such that

$$B(\widetilde{\sigma}) = R.$$

- For a risk-free bond,

$$\text{interest rate} = \max\{B, R\}$$

- l^* maximizes the return to savings:

$$\begin{aligned} B &< R \Rightarrow l^* = 0 \\ B &> R \Rightarrow l^* = 1 \end{aligned}$$

- The equilibrium saving rate is then

$$s^* = \beta^\theta (\text{return to savings})^{\theta-1}$$

where

$$\text{return to savings} = \max\{B, R\}$$

θ is the elasticity of intertemporal substitution

- If high idiosyncratic risk (sufficiently incomplete markets):

$$\begin{aligned} \sigma &> \widetilde{\sigma} \Rightarrow B < R \\ &\Rightarrow l^* = 0 \Rightarrow s^* = \beta^\theta R^{\theta-1} \end{aligned}$$

If low idiosyncratic risk (relatively complete markets):

$$\begin{aligned} \sigma &< \widetilde{\sigma} \Rightarrow B > R \\ &\Rightarrow l^* = 1 \Rightarrow s^* = \beta^\theta B^{\theta-1} \end{aligned}$$

- Risk, specialization, and savings:

	$\theta < 1$	$\theta > 1$
$\sigma > \widetilde{\sigma} \Rightarrow B < R$	$s^* = \beta^\theta R^{\theta-1} < \beta^\theta B^{\theta-1}$	$s^* = \beta^\theta R^{\theta-1} > \beta^\theta B^{\theta-1}$
$\sigma < \widetilde{\sigma} \Rightarrow B > R$	$s^* = \beta^\theta B^{\theta-1} < \beta^\theta R^{\theta-1}$	$s^* = \beta^\theta B^{\theta-1} > \beta^\theta R^{\theta-1}$

- For $\sigma < \widetilde{\sigma}$:
  - the risk-adjusted return $B = [EA^{1-\rho}]^{\frac{1}{1-\rho}}$ always falls with risk σ;
  - the saving rate $s^* = \beta^\theta B^{\theta-1}$ increases as B falls iff $\theta < 1$;
  - therefore, the saving rates increases with risk σ iff $\theta < 1$.

- Conditional on $l^* = 1$, the savings rate s^* decreases as we complete the markets iff the precautionary-savings effect is strong enough. But if the elasticity of intertemporal substitution is sufficiently high, then completing the markets raises the saving rate as it raises the risk-adjusted real return.

- Remark: If we introduce a riskless bond in zero net supply, the bond market will clear at

$$\text{interest rate} = \max\{B, R\}.$$

8.1.3 Aggregates

- For the *individual*,

$$g^i_{t+1} = \frac{y^i_{t+1}}{y^i_t} = s[R + l(A^i_{t+1} - R)].$$

If $\sigma > \widetilde{\sigma}$, $l = 0$, and $g^i_{t+1} = sR$ (non-random)

If $\sigma < \widetilde{\sigma}$, $l = 0$, and $g^i_{t+1} = sA^i_{t+1}$ (random).

- For the *aggregates*,

$$C_t = (1 - s)Y_t, \quad K_t = sY_t.$$

If $\sigma > \widetilde{\sigma}$,

$$\begin{aligned} Y_t &= RK_{t-1} \\ g^* &= s^*R = (\beta R)^\theta \end{aligned}$$

If instead $\sigma < \widetilde{\sigma}$, since idiosyncratic shocks wash out at the aggregate,

$$Y_t = \int_i (A^i_t k^i_{t-1}) = \overline{A}K_{t-1}$$

$$g^* = s^*\overline{A} = \beta^\theta B^{\theta-1}\overline{A}$$

Hence, aggregates are always deterministic.

- *Aggregate technology:*

$$\frac{Y}{K}=\begin{cases} R & \Leftrightarrow l^*=0 \Leftrightarrow B<R \Leftrightarrow \sigma>\widetilde{\sigma} \\ \overline{A} & \Leftrightarrow l^*=1 \Leftrightarrow B>R \Leftrightarrow \sigma<\widetilde{\sigma} \end{cases}$$

8.1.4 Aggregate Growth

- Let $g^o=(\beta\overline{A})^\theta$; this is the complete-markets or first-best growth rate.

- Given that $\overline{A}>R$ by assumption, and that $B<\overline{A}$ for any $\sigma>0$, we have:

	$\theta<1$	$\theta>1$
$\sigma>\widetilde{\sigma}\Leftrightarrow B<R$	$g^*=(\beta R)^\theta<g^o$	$g^*=(\beta R)^\theta<g^o$
$\sigma<\widetilde{\sigma}\Leftrightarrow B>R$	$g^*=\beta^\theta B^{\theta-1}\overline{A}>g^o$	$g^*=\beta^\theta B^{\theta-1}\overline{A}<g^o$

- Also, for $\sigma<\widetilde{\sigma}$:

$$\frac{\partial g^*}{\partial\sigma}\text{ same signs as }\frac{\partial s^*}{\partial\sigma},\text{ same signs as }1-\theta$$

- If the EIS is high, completing the markets increases savings and growth unambiguously.

- When $l^*=1$ and thus $g^*=\beta^\theta B^{\theta-1}\overline{A}$. This is **not** the growth rate $g^o=(\beta\overline{A})^\theta$ that we would calculate from a **representative agent model** with technology $Y=\overline{A}K$; nor the growth rate $g=(\beta B)^\theta$ that we would calculate from a representative agent model with technology $Y=BK$. In particular, $(\beta B)^\theta<g^*\lesseqgtr(\beta B)^\theta$. Difference due to market incompleteness. Similarly, interest rate $B<\overline{A}$, and $s^*=\beta^\theta B^{\theta-1}\neq\beta^\theta\overline{A}^{\theta-1}=s^o$.

Proposition 28 *For any $\overline{A}>R$, there is $\widetilde{\sigma}=\widetilde{\sigma}(\overline{A},R,\rho)>0$ with $\partial\widetilde{\sigma}/\partial\overline{A}>0>\partial\widetilde{\sigma}/\partial R,\partial\widetilde{\sigma}/\partial\rho$, such that*

$$\begin{aligned}\sigma &> \widetilde{\sigma}\Rightarrow\begin{Bmatrix} l^*=0,\ s^*=\beta^\theta R^{\theta-1}\lesseqgtr s^o \\ g^*=(\beta R)^\theta<g^o\end{Bmatrix} \\ \sigma &< \widetilde{\sigma}\Rightarrow\begin{Bmatrix} l^*=1,\ s^*=\beta^\theta B^{\theta-1}\lesseqgtr\beta^\theta R^{\theta-1} \\ g^*=\beta^\theta B^{\theta-1}\overline{A}>(\beta R)^\theta, g^*\lesseqgtr g^o\end{Bmatrix}\end{aligned}$$

Show Figure 1.

- The competitive equilibrium is not first-best. However, it is constrained Pareto efficient!

8.1.5 Comparison: Complete Markets vs. Financial Autarchy.

- Assume access to a complete assets market; **fully insure** against all idiosyncratic risk $\Rightarrow$ a net-of-hedging safe return $\overline{A}$.

- Since $\overline{A} > R$, specialization $l_t^i = 1 \; \forall t, i$.

- The *representative-agent model* applies and the Euler condition writes

$$U'(c_t^i) = \beta \overline{A} U'(c_{t+1}^i)$$

- *The Arrow-Debreu equilibrium:* For all i, t it holds that

$$\begin{aligned} y_t^i &= \overline{A} k_t^i, \, k_t^i = s y_t^i, \; c_t^i = (1-s) y_t^i \\ g_t^i &= s \overline{A} = (\beta \overline{A})^\theta, \quad s = \beta^\theta \overline{A}^{\theta - 1} \end{aligned}$$

- We can thus summarize:

Proposition 29 *If intertemporal substitution is strong ($\theta > 1$), then both the growth rate and the savings rate are higher under complete markets than under financial autarchy. If instead risk intertemporal substitution is weak ($\theta < 1$), then the savings rate is lower under complete markets, and the growth rate may be either higher or lower. If idiosyncratic risk had been sufficiently high (so that $B < R$), then completing the markets unambiguously raises the growth rate, whatever θ. But if idiosyncratic risk had been rather small (so that $B > R$),* *and intertemporal substituiton weak ($\theta < 1$), then and only then completing the markets can slow down growth. Finally, the interest rate is unambiguously increasing with market completeness.*

	$\theta < 1$	$\theta > 1$
$\sigma > \widetilde{\sigma} \Rightarrow B < R$	$s^* > s^o$, $g^* < g^o$	$s^* < s^o$, $g^* < g^o$
$\sigma < \widetilde{\sigma} \Rightarrow B > R$	$s^* > s^o$, $g^* > g^o$	$s^* < s^o$, $g^* < g^o$

8.1.6 The Process of Financial and Economic Development: Non-monotonicity in Growth Rates.

- *Stage I:* Highly incomplete markets, too much uninsurable idiosyncratic risk, $\sigma > \widetilde{\sigma}$. In this stage, $B < R < \overline{A}$ and $l = 0$.

- *Stage II:* Moderately incomplete markets, sufficiently low uninsurable idiosyncratic risk, $0 < \sigma < \widetilde{\sigma}$. In this intermediate stage, $\overline{A} > B > R$ and $l = 1$.

- *Stage III:* Complete financial markets, fully insured idiosyncratic risk, $\sigma \approx 0$. In this final stage, the Arrow-Debreu equilibrium applies, $B \approx \overline{A}$ and $l = 1$.

- Empirical implications? Cross-country interpretation? Time-series interpretation?

8.1.7 Growth and Income Distribution: a Kuznets Curve.

- Stage I: low growth and low income dispersion, for nobody takes risks.

- Stage II: output levels and growth rates unambiguously increase, but income dispersion raises as well, for entrepreneurs now take significant uninsurable idiosyncratic risk.

- Stage III, more and more of the idiosyncratic risk is insured away, and thus income dispersion falls, due to sufficient risk-sharing.

- A inverted-U shaped relation b/ income inequality and market sophistication $\Longrightarrow$ **a Kuznets curve**.

8.1.8 Progressive Taxation and Social Security as Insurance.

- A rational for progressive taxation, or social security: provide insurance, effectively substitute for missing markets.

- Progressive taxation may enhance growth if markets are incomplete.

The optimal tax schedule w/o aggregate uncertainty.

- Let $T_t^i(.)$ be tax payments individual i makes at t.

- To implement the Arrow-Debreu allocation after taxes,

$$\begin{aligned} u'(c_t^i) &= \beta E\left[\left[A_{t+1}^i - \frac{\partial T_t^i(.)}{\partial k_t^i}\right] u'(c_{t+1}^i)\right] \\ u'(c_t^i) &= \beta \overline{A} u'(c_{t+1}^i) \end{aligned}$$

- Optimal taxation is

$$T_t^i(.) = [A_t^i - \overline{A}]k_{t-1}^i = y_t^i - \frac{Y_t}{K_t} k_{t-1}^i$$

Ensures a certain income level $\overline{A}k_t^i$ and a certain capital return $\left[A_{t+1}^i - \frac{\partial T_t^i(.)}{\partial k_t^i}\right] = \overline{A}$ in all states.

The optimal tax schedule in the presence of aggregate fluctuations.

- Allow for exogenous aggregate fluctuations $\widetilde{A}_t$.

  $A_t^i = \widetilde{A}_t + \varepsilon_t^i$; ε_t^i i.i.d. and independent of $\widetilde{A}_t$.

  $\widetilde{A}_t$ a stationary process bounded from below by R.

- The stochastic optimal tax system:

$$T_t^i(.) = [A_t^i - \widetilde{A}_t]k_{t-1}^i = y_t^i - \frac{Y_t}{K_t}k_{t-1}^i$$

- Countercyclical taxes:

$$Corr_{t-1}(T_t^i, Y_t) = Corr_{t-1}(T_t^i, \widetilde{A}_t) = -1 < 0$$

BUT:

- The above tax implications presume government can observe idiosyncratic shocks A_t^i.

- Why should the government be able to do so, and the market not?

- What is the shocks are private information to the agents?