14.06 Midterm 2005 - Solutions

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Note: Most questions ask for an explantion. In many cases, you lost points because the explanation was missing or incomplete.

Question 1

(a) The resource constraint is

$$\dot{k}(t) = k(t)^{\alpha-\eta} - c(t) - \delta k(t)$$

To derive the Euler equation, solve the social planners's maximization problem

$$H = e^{-\rho t} \log c(t) + \mu(t)[k(t)^{\alpha-\eta} - c(t) - \delta k(t)]$$

The FOCs are

$$\begin{aligned} H_c &= e^{-\rho t} c(t)^{-1} - \mu(t) = 0 \\ H_k &= \mu(t)[(\alpha-\eta)k(t)^{\alpha-\eta-1} - \delta] = -\dot{\mu}(t) \end{aligned}$$

Now use the FOC for consumption to find $\mu(t)$ and $\mu(t)$

$$\begin{aligned} \mu(t) &= e^{-\rho t} c(t)^{-1} \\ \dot{\mu}(t) &= -\rho e^{-\rho t} c(t)^{-1} - \gamma e^{-\rho t} c(t)^{-1} \frac{\dot{c}(t)}{c(t)} \end{aligned}$$

Plugging these into the FOC for capital gives the Euler equation

$$\frac{\dot{c}(t)}{c(t)} = (\alpha-\eta)k(t)^{\alpha-\eta-1} - \delta - \rho$$

The Phase diagram is a usual, where the $\dot{c} = 0$ locus and the $\dot{k} = 0$ locus are given by

$$\begin{aligned} \dot{c} &= 0 <=> k^* = \left(\frac{\alpha-\eta}{\delta+\rho}\right)^{\frac{1}{1-\alpha+\eta}} \\ \dot{k} &= 0 <=> c(t) = k(t)^{\alpha-\eta} - \delta k(t) \end{aligned}$$

(b) Assume that $k > 1$ for simplicity. Then, as η decreases, the $\dot{c} = 0$ locus shifts to the right and the $\dot{k} = 0$ locus shifts up. Lowering η increases the returns to capital, which leads to more capital accumulation and shifts out the $\dot{c} = 0$ locus. Also, lowering η increases production for any level of k, which causes the to $\dot{k} = 0$ (the resource constraint) to shift up.

(c) It is uncertain wether c increases or decreases initially. If the wealth effect dominates, c increases. If the substitution effect dominates, c decreases. Which of these effects dominates depends on the EIS. However, we know that c has to jump onto the new saddle path (if the new saddle path goes through the old stead state, then c doesn't have to jump at all). From then on, c follows the new saddle path and both c and k smoothly converge to the new steady state.

(d) We know that the Euler equation for the household is like the one for the social planner, just with the interest rate in the place of the marginal product of capital

$$\frac{\dot{c}(t)}{c(t)} = r(t) - \delta - \rho$$

To find the interest rate, we solve the firm's maximization problem, where the firms don't take into account the externality

$$\max_{k(t)} Qk(t)^{\alpha} - r(t)k(t)$$

The FOC is

$$r(t) = \alpha Qk(t)^{\alpha-1} = \alpha k^{\alpha-1-\eta}$$

Thus we get

$$\frac{\dot{c}(t)}{c(t)} = \alpha k^{\alpha-1-\eta} - \delta - \rho$$

First note that the $\dot{k} = 0$ locus is not different in the decentralized equilibrium since the overall amount of resources for a given k does not change. However, the incentives for accumulating capital change since the private return to capital is higher. From the Euler equation, we can derive that the steady state k is now higher, which means that the $\dot{c} = 0$ locus is further to the right.

(e) The social planner's allocation is efficient, but the decentralized allocation is not. Households overinvest in capital since they don't internalize the externality. The private return to capital is higher than the social return. To restore efficiency, we could impose a distortive/proportional tax on capital income. This would give the following Euler condition

$$\frac{\dot{c}(t)}{c(t)} = [(1-\tau)\alpha k(t)^{\alpha-\eta-1} - \delta - \rho]$$

The tax that sets $(1-\tau)\alpha = (\alpha - \eta)$ is $\tau = \frac{\eta}{\alpha}$.

(f) The answer is the same as for part (b), except that the $\dot{c} = 0$ locus does not shift quite as much.

Question 2

(a) False. The neoclassical model can explain a big fraction of income differentials if we include human capital in the model (see Mankiw, Romer, Weil). However, it does not explain all of the variation. Moreover, although the model can explain conditional convergence, it predicts that all countries grow at the same rate once they are in steady state (unless technology growth varies across countries, which is not explained by the model).

(b) False. In the Ramsey model, output per capita grows at the rate of technological progress in the long-run (in steady state). Hence, in Ramsey the long-run growth rate does not depend on the EIS. However, in the AK model, the growth rate is always $\frac{1}{\theta}[A - \delta - \rho]$, where $\frac{1}{\theta}$ is the EIS.

(c) False. The immediate impact on consumption and labor supply is uncertain since it depends on whether the wealth or the substitution dominates, which in turn depends on the EIS. The increase in productivity causes the wage and the interest rate to increase, which implies that household income increases. The wealth effect leads households to consume more and to work less since they are richer. The higher wage makes leisure more expensive and gives an incentive to households to work more. The higher interest rate makes consumption today more expensive (in terms of consumption tomorrow), which leads to a decrease in consumption.

(b) False. When financial markets are incomplete, agents can not diversify all the risk they face. Since they are risk averse, they don't like investing in undiversified risky projects, which lowers savings and growth.