

14.06 Midterm

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You have 1.5 hours. Answer all questions. No books or notes are allowed. Good luck!

Question 1 [60 points/100]

Consider the following version of the neoclassical growth model, in *continuous time*.

There is no exogenous technological change and no population growth. Physical capital is accumulated in the usual way:

$$\dot{k} = i - \delta k, \quad (1)$$

where k denotes capital, i denotes investment, and $\delta > 0$ is the depreciation rate. The resource constraint of the economy is also the usual one:

$$c + i = y, \quad (2)$$

where c denotes consumption and y denotes output.

Output in turn is given by a Cobb-Douglas technology of the following form:

$$y = Qk^\alpha \quad (3)$$

where $\alpha \in (0, 1)$ and Q represents the quality of the natural environment. The latter depends negatively on the amount of capital in the economy, because more factories (higher k) increase pollution or otherwise deteriorate the natural environment:

$$Q = k^{-\eta}, \quad (4)$$

where $\eta \in [0, \alpha)$.

Finally, household preferences are logarithmic:

$$U_0 = \int_0^\infty e^{-\rho t} u(c) dt, \quad (5)$$

where $u(c) = \log c$ and $\rho > 0$.

Part I Consider first the **Pareto efficient allocations** of this economy: the social planner maximizes (5) subject to (1)-(4), given some initial capital stock $k_0 > 0$.

(a) Combine conditions (1)-(4) to derive a single equation for the resource constraint of the economy, namely an equation for \dot{k} as a function of k , c , and the parameters (α, η, δ) . Next, derive the Euler condition that characterizes the optimal consumption-saving plan of the social planner. Finally, use the two equations to draw the phase diagram of the economy in the (c, k) space.

(b) Compare the case that $\eta = \eta_H$ with the case that $\eta = \eta_L$, for some $\eta_L < \eta_H$. Show the effect in the phase diagram: how do the $\dot{c} = 0$ and $\dot{k} = 0$ loci change as η decreases? Explain.

(c) Suppose that the economy was initially (at $t = 0$) in the steady state with $\eta = \eta_H$. At that time, a technological revolution occurs – or a new environmental policy is passed – that reduces pollution. As a result, η falls permanently from η_H to η_L . Does optimal consumption at $t = 0$ increase or decrease in response to this change in η ? Explain. And what are the dynamics of consumption and capital following this change?

(Question 1 continued)

Part II Now consider the **competitive equilibrium** of the economy: individual agents do not internalize the externality involved in Q .

(d) Fix a value for $\eta > 0$. How does the phase diagram representing the competitive equilibrium compares with that for the planner's optimal allocation? Is the \dot{k} locus different? Is the \dot{c} locus different? Explain.

(e) Are the equilibrium allocations efficient? If yes, why yes? If not, why not, and what is then the kind of tax/subsidy policy that could restore efficiency?

(f) Reconsider the comparative-statics exercise of part (b) above in the context of the competitive equilibrium. Does equilibrium consumption at $t = 0$ increase or decrease in response to a decrease in η ? Explain why your answer is the same as, or different from, the one you gave for part (b).

Question 2 [40 points/100]

True, false, or uncertain? Provide a brief explanation for your answer.

(a) The neoclassical growth (Solow/Ramsey/etc.) model can well explain the observed income and growth differentials across countries.

(b) The elasticity of intertemporal substitution has no effect on the long-run growth rate of the economy in either the Ramsey model or endogenous-growth AK -type models.

(c) A positive shock in aggregate productivity necessarily increases consumption and labor supply.

(d) If agents are risk averse, incomplete financial markets can have no effect on savings and growth.

Good Luck!