Crises and Prices

Information Aggregation, Multiplicity and Volatility

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Volatility

- volatility
  - fundamentals
  - non-fundamentals
- crises
  - currency attacks
  - bank runs
  - financial crises
  - riots

Coordination: attacking regime optimal when enough agents attack

Multiple Equilibria

- multiple equilibria $\longrightarrow$ sunspot volatility
- incomplete theories?
- Morris and Shin (1998)

  perturbation away from common knowledge

  $\longrightarrow$ disperse information $\longrightarrow$ unique equilibrium
- unintended consequence: kills volatility

This Paper

- information structure role in volatility
- endogenize public info
  - Model I: financial price
  - Model II: signal of aggregate activity
- Grossman-Stiglitz meets Morris-Shin
  - information aggregated endogenously
  - noise avoids common knowledge

Main Results

better private info $\longrightarrow$ better public info

less noise $\longrightarrow$ more volatility

Main Results

- less noise $\longrightarrow$ more volatility
  - comparative static of unique equilibrium
  - introducing multiplicity
- uniqueness not perturbation
- types of multiplicity
  - regime outcome
  - asset demand and price

Related Literature

- Morris and Shin (1998, 2000)
- Atkeson (2000)
- Hellwig, Mukherji, and Tsyvinski (2004)
- Tarashev (2003)
- Angeletos, Hellwig and Pavan (2003, 2004), Edmond (2004)
- Chari and Kehoe (2004)
- Grossman and Stiglitz (1976), Barlevy and Veronesi (2004)

Basic Model

- agents $i \in [0,1]$ choose whether to "attack a status quo"
- payoffs:

	Regime Change $(R=1)$	*Status Quo* $(R=0)$
Attack $(a_i = 1)$	$1-c$	$-c$
Not $(a_i = 0)$	0	0

- A mass of agents attacking
- θ strength of status quo
- status quo abandoned $\quad R = 1 \iff A > \theta$

Common Knowledge

- let $\underline{\theta} \equiv 0$ and $\overline{\theta} \equiv 1$
- with common knowledge of θ:

$$\theta \leq \underline{\theta} \quad \longrightarrow \quad A = 1 \quad R = 1$$

$$\theta > \overline{\theta} \quad \longrightarrow \quad A = 0 \quad R = 0$$

$$\theta \in (\underline{\theta}, \overline{\theta}] \quad \longrightarrow \quad \text{multiple equilibria}$$

Morris-Shin (Exogenous Information)

- private signal:

$$x_i = \theta + \xi_i \qquad \xi_i \sim \mathcal{N}(0, \sigma_x^2)$$

Proposition. equilibrium unique

Morris-Shin (Exogenous Information)

- private signal:

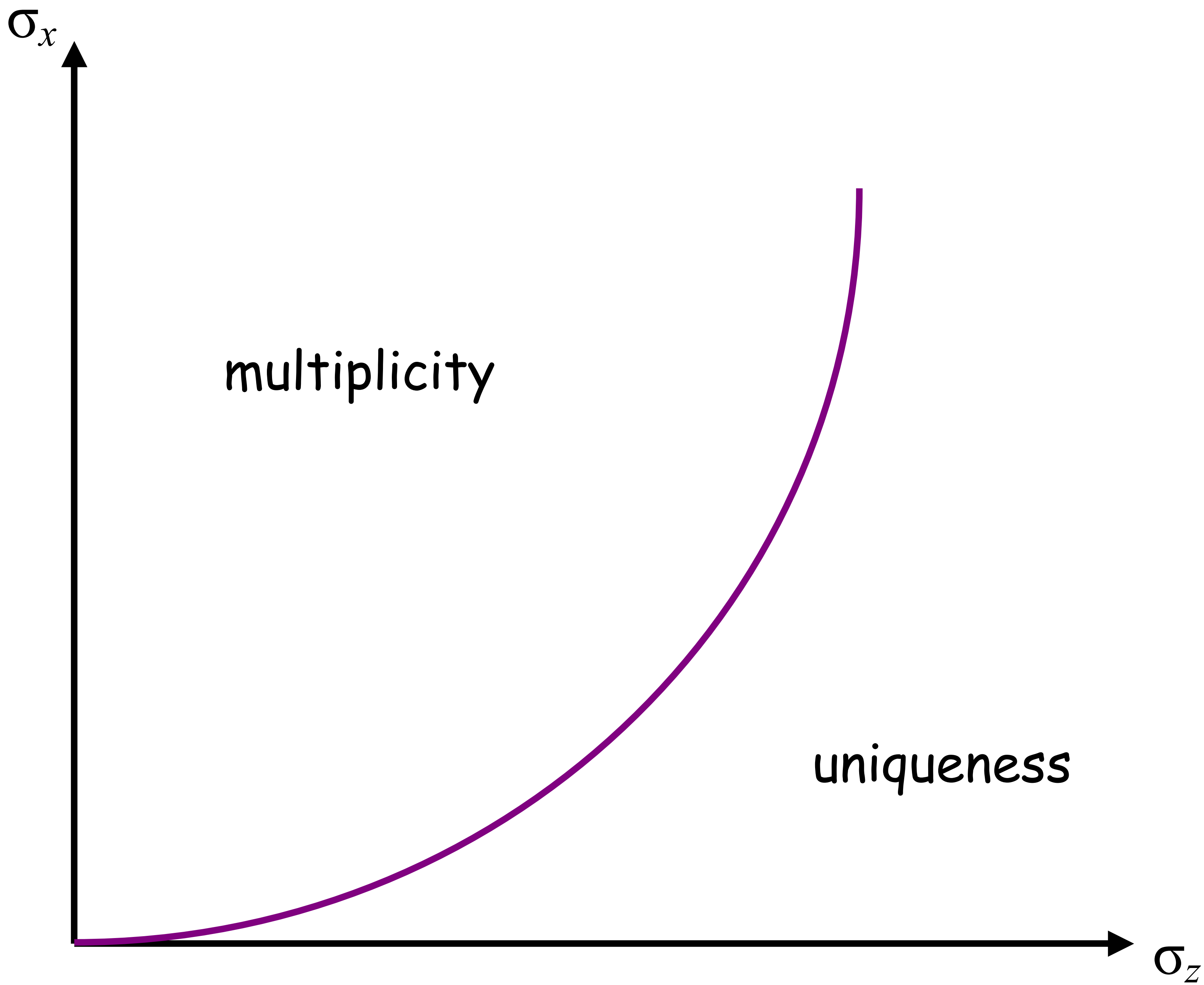
$$x_i = \theta + \xi_i \qquad \xi_i \sim \mathcal{N}(0, \sigma_x^2)$$

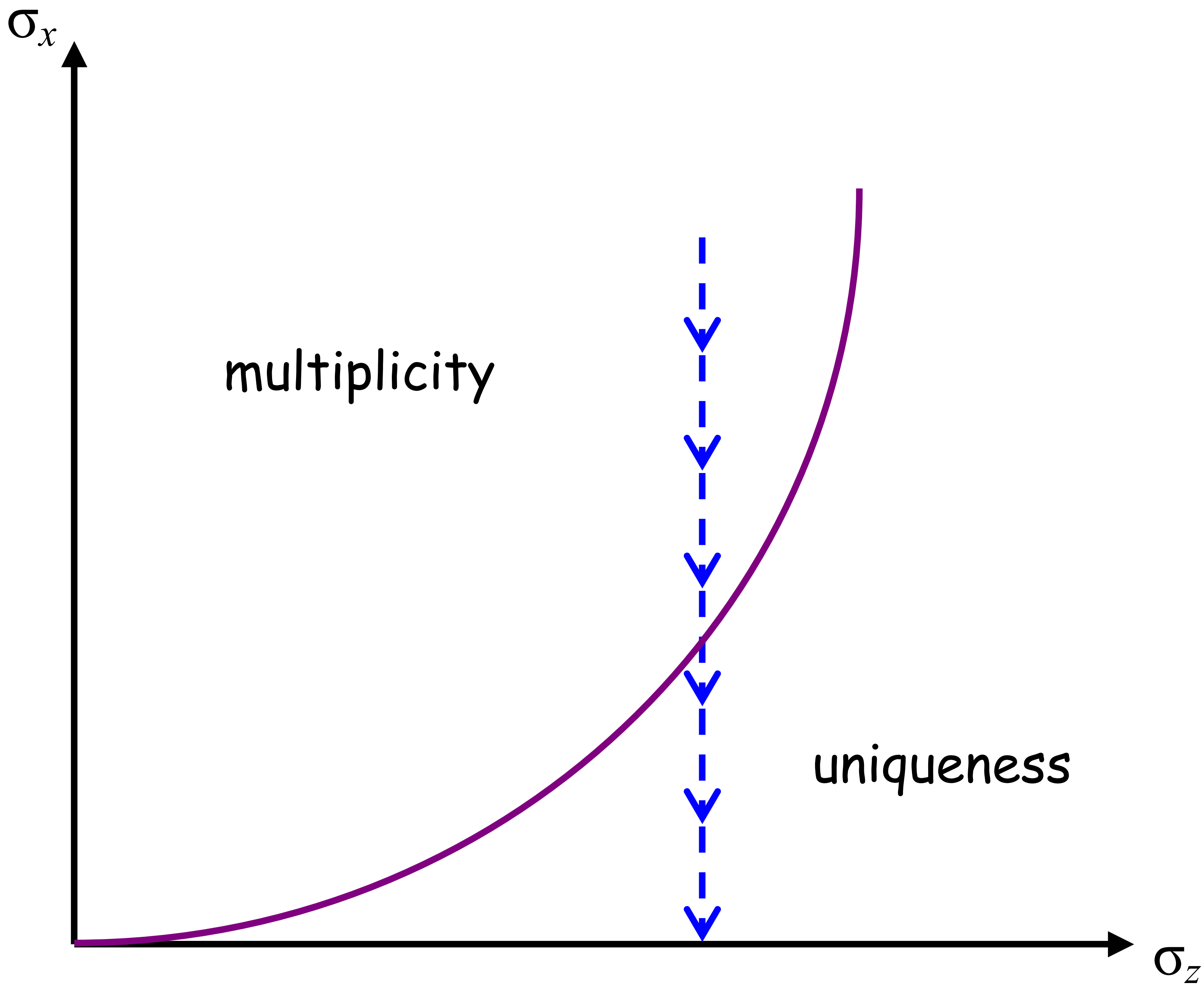
- public signal:

$$z = \theta + v \qquad v \sim \mathcal{N}(0, \sigma_z^2)$$

Proposition. equilibrium unique iff

$$\frac{\sigma_x}{\sigma_z^2} \leq \sqrt{2\pi}$$


σ_x
multiplicity
uniqueness
σ_z



σ_x
multiplicity
uniqueness
σ_z

Missing

- missing...
  - prices or other endogenous indicators
  - some knowledge about others actions
- public info
  - bank runs: deposit information
  - currency crises: peso forward
  - riots: attendance reported

$\Rightarrow$ public information largely endogenous

Model I: Financial Prices

stage 1: financial market

- agents trade financial asset

stage 2: coordination game

- use information revealed by price
- agents attack or not

Financial Market

- risky asset: price p, dividend f
- random supply

$$\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- payoff from financial trade:

$$V(k_i,\, f,\, p) = u\left(w_i - pk_i + fk_i\right)$$

$$u(c) = -\exp(-\gamma c)/\gamma$$

- two cases for dividend:

exogenous $f = f(\theta)$ vs. endogenous $f = f(A)$

Equilibrium Definition

Equilibrium: functions (P, k, K, a, A) such that

stage 1

$$p = P(\theta, \varepsilon)$$

$$k(x,p) = \arg\max_{k\in\mathbb{R}} \mathbb{E}\left[\, V(k, f, p) \mid x, p \,\right]$$

$$K(\theta, p) = \int_x k(x,p) d\Phi\left(\frac{x-\theta}{\sigma_x}\right)$$

$$K(\theta, p) = \varepsilon$$

stage 2

$$a(x,p) = \arg\max_{a\in[0,1]} \mathbb{E}\left[\, U(a, R) \mid x, p \,\right]$$

$$A(\theta, p) = \int_x a(x,p) d\Phi\left(\frac{x-\theta}{\sigma_x}\right)$$

(1) Exogenous Dividend

- dividend

$$f = f(\theta) = \theta$$

- optimal k

$$k = \frac{\mathbb{E}[\theta|x,p] - p}{\gamma \mathrm{Var}[\theta|x,p]}$$

- guess and verify

$$\mathbb{E}[\theta|x,p] = \delta x + (1-\delta)p$$

$$\mathrm{Var}[\theta|x,p] = \sigma^2$$

(1) Exogenous Dividend

- aggregate demand

$$K(\theta, p) \;=\; \frac{\delta(\theta - p)}{\gamma\sigma^2}$$

- market clearing

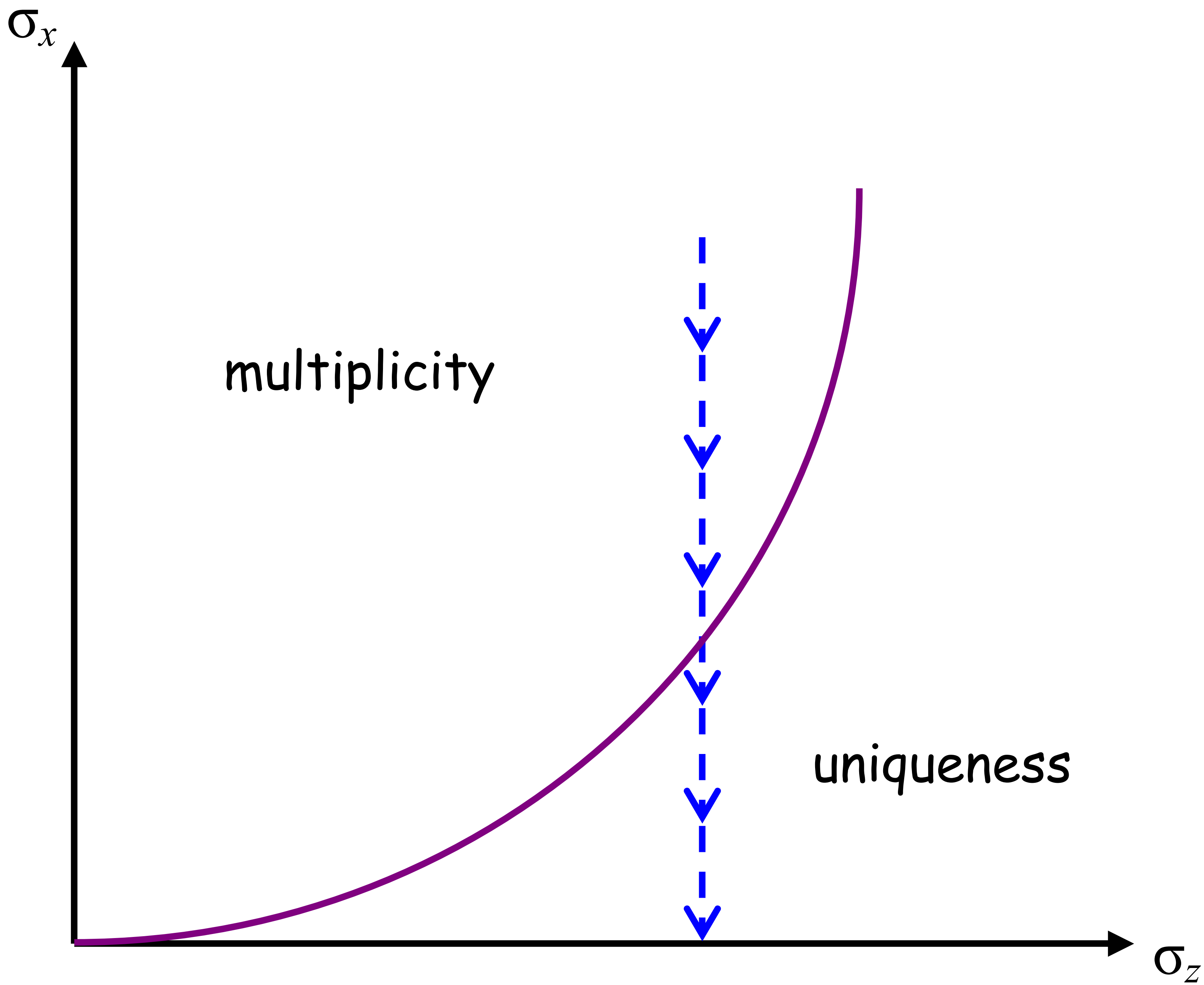
$$K(\theta, p) \;=\; \varepsilon \quad\Leftrightarrow\quad p \;=\; P(\theta, \varepsilon) \;=\; \theta - \frac{\gamma\sigma^2}{\delta}\varepsilon$$

- price $\longrightarrow$ public signal with

$$\sigma_z \;=\; \frac{\gamma\sigma^2}{\delta}\sigma_\varepsilon$$

- Normality $\longrightarrow$ compute $\delta,\ \sigma,\ \sigma_z$

$$\sigma_z = \gamma \sigma_\varepsilon \sigma_x^2$$


σ_x
multiplicity
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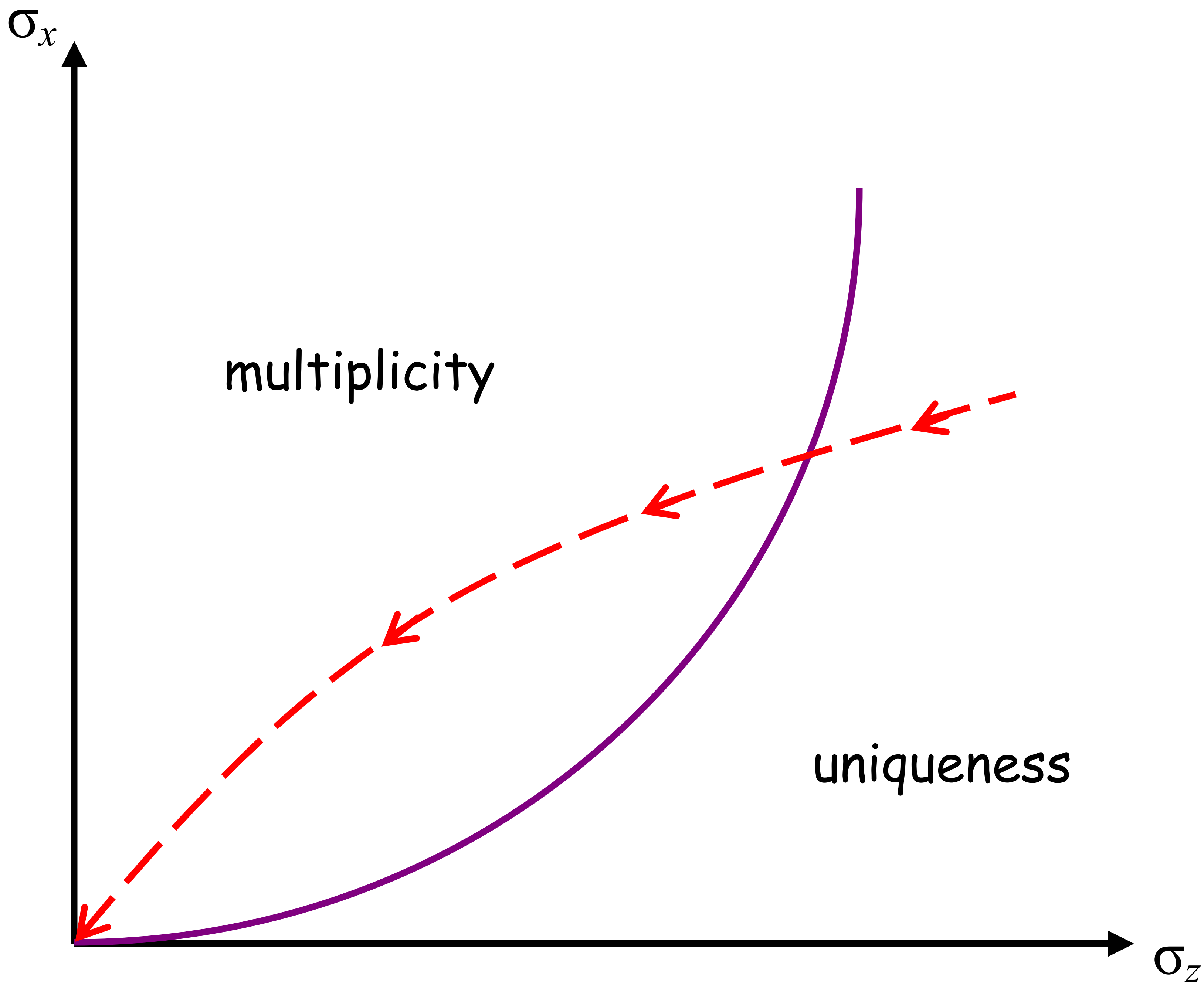
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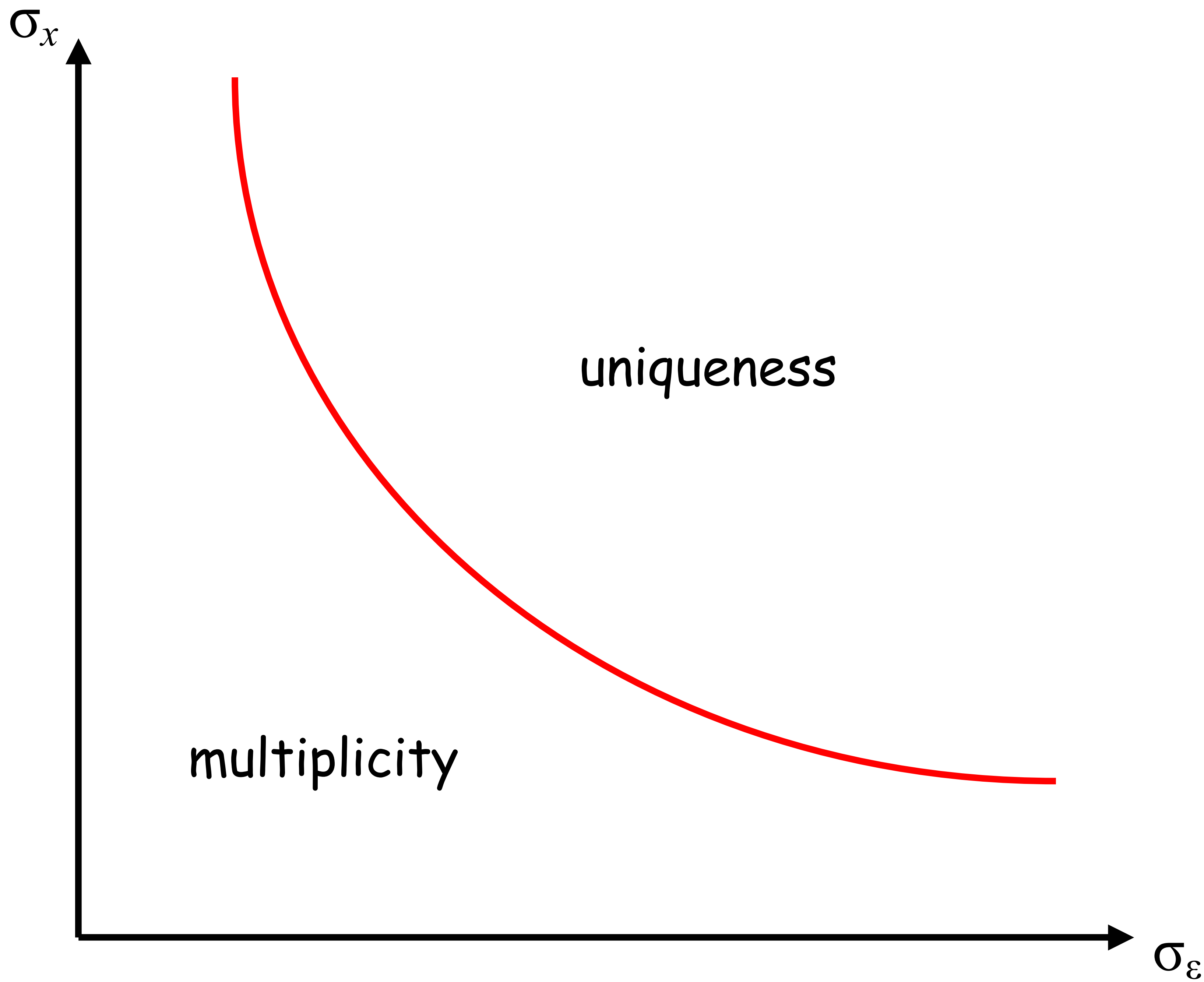


σ_x
uniqueness
multiplicity
σ_ε

(1) Exogenous Dividend

Proposition. multiple equilibria iff

$$\sigma_\varepsilon^2 \sigma_x^3 < \gamma^2 (2\pi)^{-1/2}$$

multiplicity in (a, A); unique (P, k, K)

- better private info $\longrightarrow$ better public info
- small noise $\longrightarrow$ multiple equilibria
- large noise $\longrightarrow$ unique equilibrium
- no perturbation argument for uniqueness

(2) Endogenous Dividend

- same setup but endogenous dividend

$$f \;=\; f(A)$$

- $f(A) = -\Phi^{-1}(A) \rightarrow$ Normality
- info revealed by price

$$\sigma_z = \gamma \sigma_\varepsilon \sigma_x^3$$

(2) Endogenous Dividend

Proposition: multiple equilibria iff $\sigma_\varepsilon^2 \sigma_x^5 < \gamma^2 (2\pi)^{-1/2}$

- uniqueness in attack $a(x,p)$ and demand $k(x,p)$
- multiplicity in price $P(\theta, \varepsilon)$

(3) Other Cases

- noisy return – *exogenous* dividend
  - multiplicity only in (A, R), not in (P, K)
  - multiplicity when σ_ε small...
  - ... but uniqueness when σ_x small

- noisy return – *endogenous* dividend
  - multiplicity when either σ_ε or σ_x small
  - multiplicity in price

Model II: Observable Actions

- private signal:

$$x_i = \theta + \xi_i \qquad \xi_i \sim \mathcal{N}(0, \sigma_x^2)$$

- public signal:

$$y = s(A, \varepsilon) \qquad \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- $\sigma_x, \sigma_\varepsilon$ exogenous $\longrightarrow$ σ_z endogenous

Model II: Observable Actions

- private signal:

$$x_i = \theta + \xi_i \qquad \xi_i \sim \mathcal{N}(0, \sigma_x^2)$$

- public signal:

$$y = s(A, \varepsilon) \qquad \varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$$

- $\sigma_x, \sigma_\varepsilon$ exogenous $\longrightarrow$ σ_z endogenous
- to preserve normality (Dasgupta, 2002):

$$s(A, \varepsilon) = \Phi^{-1}(A) + \varepsilon$$

Equilibrium Definition

Equilibrium: functions (Y, a, A) such that

$$
\begin{aligned}
y &= Y(\theta, \varepsilon) \\
a(x, y) &= \arg\max_{a \in [0,1]} \mathbb{E}[\, U(a, R) \mid x,\; y \,] \\
A(\theta, y) &= \int_x a(x, y) d\Phi\left(\frac{x-\theta}{\sigma_x}\right) \\
y &= s(\; A(\theta, y) \;,\; \varepsilon \;)
\end{aligned}
$$

where $R = 1$ iff $A(\theta, y) \geq \theta$

Equilibrium Analysis

- *monotone* equilibrium: thresholds x^* and θ^* such that
  - agent attacks iff $x \leq x^*(y)$
  - status quo abandoned iff $\theta \leq \theta^*(y)$
- four steps:
  1. (aggregation) $x^* \rightarrow A,\ \theta^*, Y$
  2. (optimality) θ^*,$Y \rightarrow x^{**}$
  3. (fixed point) $x^{**} = x^*$
  4. determinacy of Y

Step 1 (Aggregation)

- aggregation: $x^* \to A,\ \theta^*, Y$
- equilibrium A :

$$A(\theta, y) = \Phi\left(\frac{x^*(y) - \theta}{\sigma_x}\right)$$

- threshold θ^*

$$A(\theta^*(y), y) = \theta^*(y)$$

$$\Updownarrow$$

$$\boxed{x^*(y) = \theta^*(y) + \sigma_x \Phi^{-1}\left[\theta^*(y)\right]}$$

Step 1 (Aggregation)

- solving for Y...

$$y = \Phi^{-1}(A(\theta, y)) + \varepsilon$$

$$\Updownarrow$$

$$\boxed{x^*(y) - \sigma_x y = \theta - \sigma_x \varepsilon}$$

- define correspondence:

$$\mathcal{Y}(z) = \{\ y \in \mathbb{R} \mid x^*(y) - \sigma_x y = z\ \}$$

- $Y(\theta, \varepsilon) \in \mathcal{Y}(\theta - \sigma_x \varepsilon)$

(Information)

- observation of y equivalent to observation of

$$z = Z(y) \equiv x^*(y) - \sigma_x y = \theta - \sigma_x \varepsilon$$

$\longrightarrow$ public signal with noise $\mathcal{N}(0, \sigma_x^2 \sigma_\varepsilon^2)$

- posterior is Normal

$$\sigma_z = \sigma_x \sigma_\varepsilon$$

Step 2 (Optimality)

- optimality: $\theta^*, Y \to x^{**}$
- equilibrium payoff:

$$a = 1 \;\to\; \mathrm{E}U = \Pr\left[\,\theta \leq \theta^*(y) \mid x, y\,\right] - c$$

- threshold x^{**}

$$\Pr\left[\theta \leq \theta^*(y) \mid x^{**}(y), y\right] = c$$

$$\Updownarrow \text{ (normality)}$$

$$\boxed{1 - \Phi\left(\frac{1}{\sigma}\left[\delta x^{**}(y) + (1-\delta)\left(x^*(y) - \sigma_x y\right) - \theta^*(y)\right]\right) = c}$$

Step 3 (Fixed Point)

- fixed point: $x^{**} = x^*$
- combining ...
- ... unique equilibrium thresholds x^* and θ^* :

$$\begin{aligned} \theta^*(y) &= \Phi\left[\frac{1}{1+\sigma_\varepsilon^2}y + \sqrt{\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}}\Phi^{-1}(1-c)\right] \\ x^*(y) &= \theta^*(y) + \sigma_x \Phi^{-1}[\theta^*(y)] \end{aligned}$$

Step 4 (Determinacy of Y)

- solving for the equilibrium signal Y...

$$\boxed{x^*(y) - \sigma_x y \;=\; z}$$

$$\Updownarrow$$

$$F(y) \equiv \Phi\left(\frac{1}{1+\sigma_\varepsilon^2}y + \Lambda\right) + \sigma_x\left[-\frac{\sigma_\varepsilon^2}{1+\sigma_\varepsilon^2}y + \Lambda\right] = z$$

- $\mathcal{Y}(z)$ non-empty
- $\sigma_\varepsilon^2\sigma_x \geq (2\pi)^{-1/2} \Rightarrow \mathcal{Y}(z)$ single-valued for all z
- $\sigma_\varepsilon^2\sigma_x < (2\pi)^{-1/2} \Rightarrow \mathcal{Y}(z)$ three values for $z \in (\underline{z}, \overline{z})$

Results

Proposition. multiple equilibria iff

$$\sigma_\varepsilon^2 \sigma_x < (2\pi)^{-1/2}$$

multiplicity only in Y, not in x^* and θ^*

- small noise $\longrightarrow$ multiple equilibria
- large noise $\longrightarrow$ unique equilibrium
- no perturbation argument for uniqueness

Non-Simultaneous Signal

- avoid RE fixed point $\longrightarrow$ simple dynamics
- "early" and "late" movers
  - early move first $\longrightarrow$ only private signals
  - late move second $\longrightarrow$ also public signal $s(A_{early}, \varepsilon)$
- equilibrium game-theoretic
- similar results as before

Morris-Shin and Common-Knowledge Limits

Proposition. Morris-Shin as $\sigma_\varepsilon \to \infty$

$$R = 1 \quad \Leftrightarrow \quad \theta \leq 1 - c$$

Proposition. common-knowledge outcomes as σ_ε or $\sigma_x \to 0$

$$\Pr\left[R = 1|\theta\right] \to 0 \quad \text{for all } \theta \in (\underline{\theta}, \overline{\theta})$$

$$\Pr\left[R = 1|\theta\right] \to 1 \quad \text{for all } \theta \in (\underline{\theta}, \overline{\theta})$$

Market Volatility

- low σ_ε or σ_x may introduce multiplicity
- less noise $\longrightarrow$ sunspot volatility
- volatility maximal when either σ_ε or $\sigma_x \to 0$

Market Volatility

- uniqueness → comparative static σ_ε and σ_x
- normalize shock $\varepsilon \leftrightarrow \varepsilon/\sigma_\varepsilon$
- outcome
$$R = 1 \quad \Leftrightarrow \quad \theta \leq \hat{\theta}(\varepsilon)$$
where $\hat{\theta}(\varepsilon)$ solves
$$\theta = \theta^*(\theta, P(\theta, \varepsilon))$$
- similar for observable actions

Market Volatility

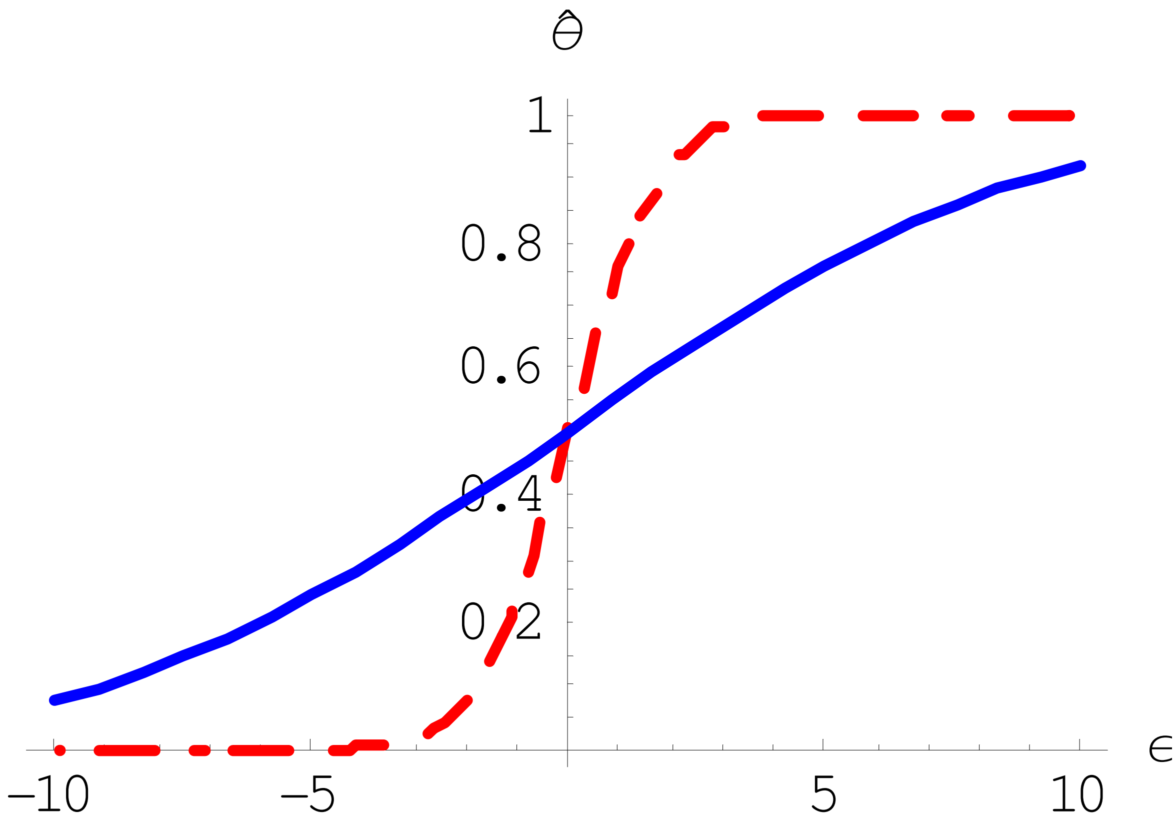
- result

$$\hat{\theta}(\varepsilon) = \Phi\left(\frac{1}{\gamma \sigma_\varepsilon \sigma_x^2} \varepsilon\right)$$

- smaller σ_ε or σ_x

  $\longrightarrow$ more sensitivity of $\hat{\theta}$ to ε

  $\longrightarrow$ more volatility



$\hat{\theta}$
1
0.8
0.6
0.4
0.2
−10
−5
5
10
ϵ

Market Volatility

- comparative statics for price volatility
- equilibrium price:

$$p = f - (\gamma \sigma_\varepsilon \sigma_x^2)\widetilde{\varepsilon}$$

- $f = f(\theta) \longrightarrow$ volatility of f exogenous
- $f = f(A) \longrightarrow$ volatility of f depends on σ_x and σ_ε

Proposition. with exogenous dividend, less noise reduces price volatility; but with endogenous dividend, less noise may increase price volatility.

Conclusions

- endogenous information:
  - indicators of aggregate activity
  - financial prices
- results:
  - better private info $\longrightarrow$ better public info
  - multiplicity when noise small
  - less noise $\longrightarrow$ more volatility
- welfare and policy implications