14.102 Problem Set 1

Due Thursday, September 22, in class

1. Lecture Notes Exercise 12: Show that $\mathbb{Q}$, the set of real rational numbers, does not have the least upper-bound property.

2. Is the set of real irrational numbers countable?

3. For $x \in \mathbb{R}^1$ and $y \in \mathbb{R}^1$, define

   (a) $d_1(x, y) = (x - y)^2$

   (b) $d_2(x, y) = |x - 2y|$

   (c) $d_3(x, y) = \frac{|x-y|}{1+|x-y|}$

   Determine for each of these whether it is a metric or not.

4. Lecture Notes Exercise 37: Prove that the only limit point of a convergent sequence is its limit.

5. Show that if a sequence $\{x_n\}$ satisfies the Cauchy criterion, then it is bounded.

6. Let E^o denote the set of all interior points of a set E (also called the interior of E).

   (a) Prove that E^o is always open.

   (b) Prove that E is open if and only if $E = E^o$.

   (c) If $G \subset E$ and G is open, prove that $G \subset E^o$.

   (d) Prove that the complement of E^o is the closure of the complement of E.

7. Let f be a continuous real function on a metric space X. Let $Z(f)$ be the set of all $p \in X$ at which $f(p) = 0$. Prove that $Z(f)$ is closed.

8. Prove that every Cobb-Douglas Function $F(x, y) = Ax^a y^b$ with A, a, and b all positive is quasiconcave.