14.102 Problem Set 3

Due Tuesday, October 18, in class

1. Lecture Notes Exercise 208: Find $\int_a^b \log(t)dt$, where $0 < a < b$ are real numbers.

2. (Sundaram 4.4, page 110) Find and classify all critical points (local maximum, local minimum, neither) of the following function: $f(x,y) = e^{2x}(x + y^2 + 2y)$. For local optima that you find figure out whether they are also global optima.

3. (Simon and Blume 18.7, page 423): Find the max and min of $f(x,y,z) = yz + xz$ subject to $y^2 + z^2 = 1$ and $xz = 3$

4. (Simon and Blume 18.11, page 434): Maximize $f(x,y) = 2y^2 - x$, subject to $x^2 + y^2 \leq 1, x \geq 0, y \geq 0$.

5. Let $F(x,y) = 2x^2 + 2y^2 + 8$ and $G(x,y) = x^2 + 2y^2 - 6x - 7$. Note: this problem will be much easier and less tedious if you stop now and think about what these functions 'look like'.

   (a) State the implicit function theorem. Find all points on the curve $G(x,y) = 0$ around which either y is not expressible as a function of x or x is not expressible as a function of y. Compute $y'(x)$ along the curve at the origin.

   (b) Find all unconstrained optima of F and G on $\mathbb{R}^2$. Is the Weierstraß theorem applicable?

   (c) Maximize and minimize $d(x,y) = \sqrt{x^2 + y^2}$ subject to $G(x,y) \leq 0$. Does Weierstraß apply?

   (d) Maximize and minimize $F(x,y)$ subject to $G(x,y) = 0$. Is the Weierstraß theorem applicable?

   (e) Maximize and minimize $F(x,y)$ subject to $G(x,y) \leq 0$. Is the Weierstraß theorem applicable?

   (f) Maximize and minimize $F(x,y)$ subject to $G(x,y) \geq 0$. Is the Weierstraß theorem applicable?

6. (Simon and Blume 20.1, page 493): Which of the following functions are homogeneous? What are the degrees of homogeneity of the homogeneous ones?

   (a) $3x^5y + 2x^2y^4 - 3x^3y^3$

   (b) $x^{1/2}y^{-1/2} + 3xy^{-1} + 7$

   (c) $x^{3/4}y^{1/4} + 6x$

(d) $\frac{(x^2-y^2)}{(x^2+y^2)} + 3$

7. (Simon and Blume 20.6, page 493): Prove that if f and g are functions on $\mathbb{R}^n$ that are homegeneous of different degrees, then $f+g$ is not homogeneous.

8. Many utility functions we work with exhibit diminishing marginal returns (i.e., they are concave in each of their arguments, $\frac{\partial^2 u}{\partial x_i^2} < 0$). Is this an ordinal property? Why or why not?

9. Show that the following functions are homothetic:

   (a) $e^{x^2 y} e^{xy^2}$

   (b) $2 \log x + 3 \log y$

   (c) $x^3 y^6 + 15x^2 y^4 + 75xy^2 + 125$