14.12 Game Theory – Midterm I
10/10/2001

Prof. Muhamet Yildiz
Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Each question is 25 points. Good luck!

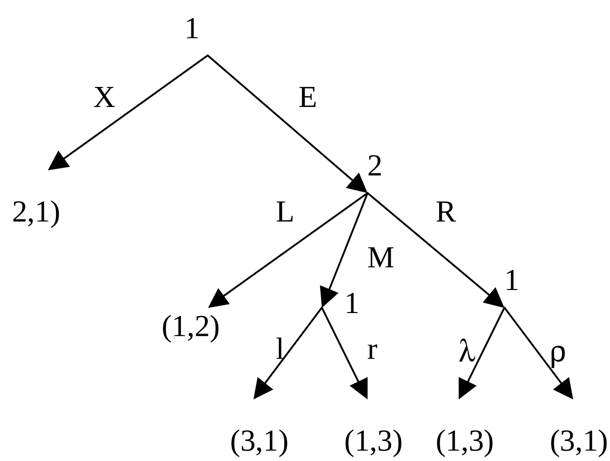
1. Find all the Nash equilibria in the following game:

1\2	L	M	R
T	1,0	0,1	5,0
B	0,2	2,1	1,0

2. Find all the pure strategies that are consistent with the common knowledge of rationality in the following game. (State the rationality/knowledge assumptions corresponding to each operation.)

1\2	L	M	R
T	1,1	0,4	2,2
M	2,4	2,1	1,2
B	1,0	0,1	0,2

3. Consider the following extensive form game.



   (a) Using Backward Induction, compute an equilibrium of this game.
   (b) Find the normal form representation of this game.
   (c) Find all pure strategy Nash equilibria.

4. In this question you are asked to compute the rationalizable strategies in linear Bertrand-duopoly with discrete prices. We consider a world where the prices must be the positive multiples of cents, i.e.,

$$P = \{0.01, 0.02, \ldots, 0.01n, \ldots\}$$

is the set of all feasible prices. For each price $p \in P$, the demand is

$$Q(p) = \max\{1 - p, 0\}.$$

We have two firms $N = \{1, 2\}$, each with zero marginal cost. Simultaneously, each firm i sets a price $p_i \in P$. Observing the prices p_1 and p_2, consumers buy from the firm with the lowest price; when the prices are equal, they divide their demand equally between the firms. Each firm i maximizes its own profit

$$\pi_i(p_1, p_2) = \begin{cases} p_i Q(p_i) & \text{if } p_i < p_j \\ p_i Q(p_i)/2 & \text{if } p_i = p_j \\ 0 & \text{otherwise,} \end{cases}$$

where $j \neq i$.

(a) Show that any price p greater than the monopoly price $p^{mon} = 0.5$ is strictly dominated by some strategy that assigns some probability $\epsilon > 0$ to the price $p^{\min} = 0.01$ and probability $1 - \epsilon$ to the price $p^{mon} = 0.5$.

(b) Iteratively eliminating all the strictly dominated strategies, show that the only rationalizable strategy for a firm is $p^{\min} = 0.01$.

(c) What are the Nash equilibria of this game?