

14.12 Game Theory – Midterm I

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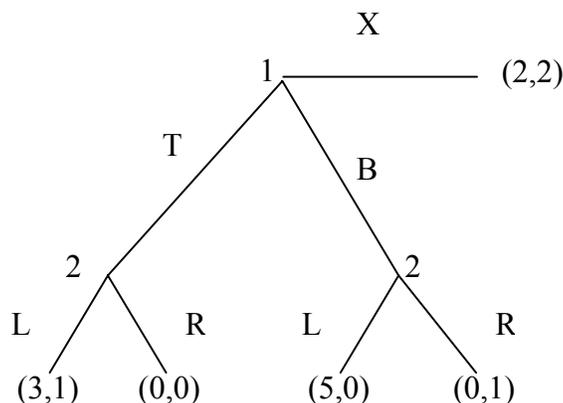
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Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Each question is 33 points. Good luck!

1. Consider the following game.

$1 \setminus 2$	L	M	R
T	3,2	4,0	1,1
M	2,0	3,3	0,0
B	1,1	0,2	2,3

- (a) Iteratively eliminate all the strictly dominated strategies.
 - (b) State the rationality/knowledge assumptions corresponding to each elimination.
 - (c) What are the rationalizable strategies?
 - (d) Find all the Nash equilibria. (Don't forget the mixed-strategy equilibrium!)
2. Consider the following extensive form game.



- (a) Find the normal form representation of this game.
 - (b) Find all pure strategy Nash equilibria.
 - (c) Which of these equilibria are subgame perfect?
3. Consider two agents $\{1, 2\}$ owning one dollar which they can use only after they divide it. Each player's utility of getting x dollar at t is $\delta^t x$ for $\delta \in (0, 1)$. Given any $n > 0$, consider the following n -period symmetric, random bargaining model. Given any date $t \in \{0, 1, \dots, n - 1\}$, we toss a fair coin; if it comes Head (which comes with probability $1/2$), we select player 1; if it comes Tail, we select player 2. The selected player makes an offer $(x, y) \in [0, 1]^2$ such that $x + y \leq 1$. Knowing what has been offered, the other player accepts or rejects the offer. If the offer (x, y) is accepted, the game ends, yielding payoff vector $(\delta^t x, \delta^t y)$. If the offer is rejected, we proceed to the next date,

when the same procedure is repeated, except for $t = n - 1$, after which the game ends, yielding $(0,0)$. The coin tosses at different dates are stochastically independent. And everything described up to here is common knowledge.

- (a) Compute the subgame perfect equilibrium for $n = 1$. What is the value of playing this game for a player? (That is, compute the expected utility of each player before the coin-toss, given that they will play the subgame-perfect equilibrium.)
- (b) Compute the subgame perfect equilibrium for $n = 2$. Compute the expected utility of each player before the first coin-toss, given that they will play the subgame-perfect equilibrium.
- (c) What is the subgame perfect equilibrium for $n \geq 3$.