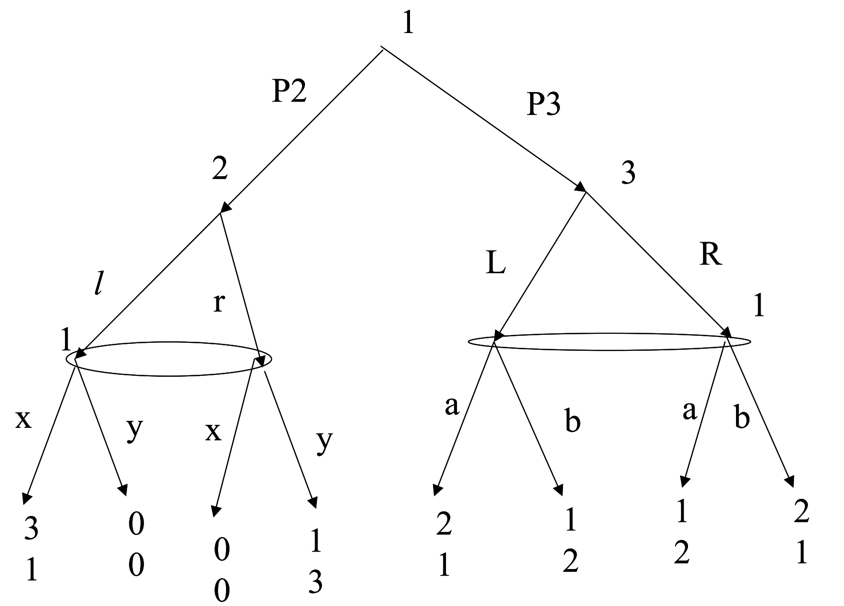
14.12 Game Theory
Fall 2002
Problem Set 3
Due on 11/4

1. Consider the following game:



   (a) Compute all subgame-perfect Nash equilibria.

   (b) Using a forward-induction argument eliminate one of these equilibria.

2. Consider an infinitly repeated version of the Bertand duoploy game. Market demand is given by $Q = 1 - P$, and each firm has constant marginal cost equal to 0. Consumers buy from the firm with the lowest price, and if there are more than one firm charging the lowest price, then demand is shared by such firms equally.

   (a) What is the minimum δ for which the monopoly price can be supported as a subgame perfect equilibrium?

   (b) What is the minimum δ when there are n firms?

3. Consider the following "repeated" Stackelberg duopoly, where a long-run firm plays against many short-run firms, each of which is in the market only for one date, while the long-run firm remains in the market throughout the game. At each date t, first, the short run firm sets its quantity x_t; then, knowing x_t, the long-run firm sets its quantity y_t; and each sells his good at price $p_t = 1 - (x_t + y_t)$. The marginal costs are all 0. The short-run firm maximizes its profit, which incurs at t. The long-run firm maximizes the present value of its profit stream where the discount rate is $\delta = 0.99$. At the beginning of each date, the actions taken previously are all common knowledge.

(a) What is the subgame perfect equilibrium if there are only finitely many dates, i.e., $t \in \{0, 1, \ldots, T\}$.

(b) Now consider the infinitely repeated game. Find a subgame perfect equilibrium, where $x_t = 1/4$ and $y_t = 1/2$ at each t on the path of equilibrium play, namely in the contingencies that happen with positive probability given the strategies.

(c) Can you find a subgame perfect equilibrium, where $x_t = y_t = 1/4$ for each t on the path of equilibrium play?

4. Assuming that the discount rate is $\delta = 0.99$, check whether the strategy profiles below are subgame-perfect Nash equilibria in the infinitely repeated game where the stage game is the following prisoners' dilemma game:

	C	D
C	5,5	0,6
D	6,0	1,1

(a) Each player always plays C.

(b) Each player always plays D.

(c) Player 1 alternates between C and D, while player 2 always plays C; if any player deviates from this scenerio, then each plays D thereafter.

(d) Each player plays C, and if a player deviates, then he plays C and the other player plays D thereafter.

(e) There are four modes, CC; P1, P2, and DD. We start in CC mode, when each player plays C. If a player i plays D in CC mode while the other plays C, then we go to Pi mode. In Pi mode, player i plays C while the other player plays D. Once we are in Pi mode we will stay in Pi mode until player i plays C five times in a row, at which point we go back to CC mode. If both players play D in CC mode, then we go to DD mode, and stay there forever. In DD mode, each plays D.