

# Homework 1

Due on 9/25/2002 (in class)

1. Consider an agent with Von Neumann and Morgenstren utility function  $u$  with  $u(x) = (x - 1)^2$ . Check whether the following VNM utility functions can represent this agent's preferences. (Provide the details.)

- (a)  $u^* : x \mapsto x - 1$ ;
- (b)  $u^{**} : x \mapsto (x - 1)^4$ ;
- (c)  $\hat{u} : x \mapsto -(x - 1)^2$ ;
- (d)  $\tilde{u} : x \mapsto 2(x - 1)^2 - 1$ .

2. Compute the set of rationalizable strategies in the following game that is played in a class of  $n$  students where  $n \geq 2$ : Without discussing with anyone, each student  $i$  is to write down a real number  $x_i \in [0, 100]$  on a paper and submit it to the TA. The TA will then compute the average

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

of these numbers. The payoff of any student  $i$  is  $100 - |x_i - 2\bar{x}/3|$  where  $x_i$  is the number student bids. Everything described above is common knowledge.

3. Consider the game depicted in Figure 1 in extensive form (where the payoff of player 1 is written on top, and the payoff of 2 is on the bottom).

- (a) Write this game in strategic form.
- (b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?
- (c) What are the Nash equilibria in pure strategies?

4. Consider the following game in normal form.

	L	M	R
S1	2,2	3,0	4,0
S2	3,3	2,0	1,0
S3	1,3	5,5	0,2
S4	1,1	1,1	2,3

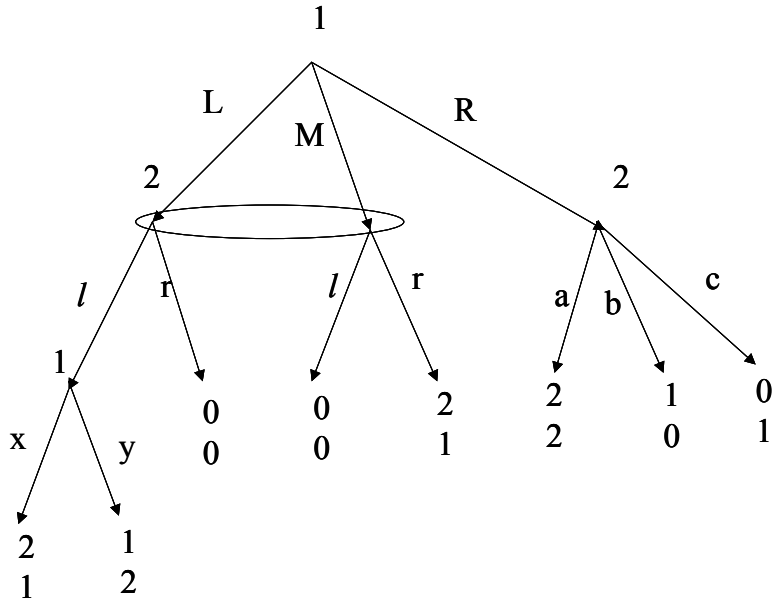


Figure 1:

- (a) Iteratively eliminate all strictly dominated strategies; state the assumptions necessary for each elimination.
  - (b) What are the rationalizable strategies?
  - (c) What are the pure-strategy Nash equilibria?
5. We have an employer and a worker. The employer provides the capital  $K$  (in terms of investment in technology, etc.) and the worker provides the labor  $L$  (in terms of the investment in the human capital) to produce  $f(K, L) = \sqrt{KL}$ , which they share equally. The parties determine their investment level (the employer's capital  $K$  and the worker's labor  $L$ ) simultaneously. The per-unit costs of capital and the labor for the employer and the worker are  $r > 1/4$  and  $c > 1/4$ , respectively. The worker cannot invest more than  $\bar{L}$  which is positive. The payoffs for the employer and the worker are

$$\frac{1}{2}f(K, L) - rK$$

and

$$\frac{1}{2}f(K, L) - cL,$$

respectively. Everything described up to here is common knowledge. Find all the rationalizable strategies.