

Lectures 10 -11

Repeated Games

14.12 Game Theory
Muhamet Yildiz

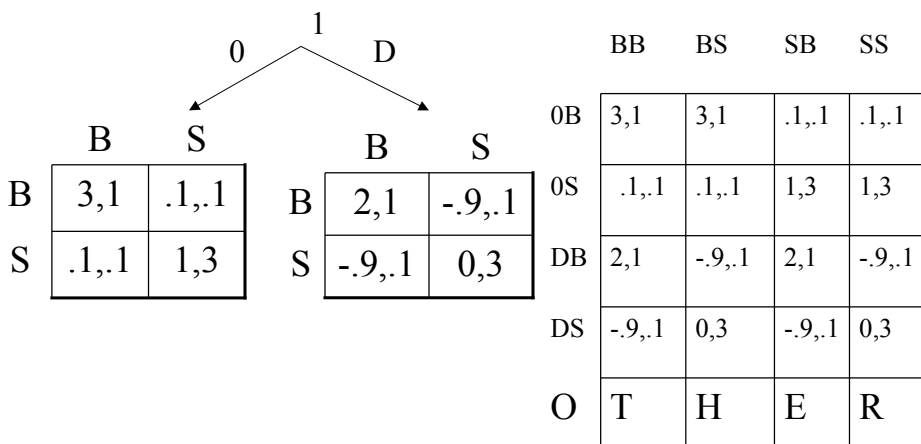
Road Map

1. Forward Induction – Examples
2. Finitely Repeated Games with observable actions
 1. Entry-Deterrence/Chain-store paradox
 2. Repeated Prisoners' Dilemma
 3. A general result
 4. When there are multiple equilibria
3. Infinitely repeated games with observable actions
 1. Discounting / Present value
 2. Examples
 3. The Folk Theorem
 4. Repeated Prisoners' Dilemma, revisited –tit for tat
 5. Repeated Cournot oligopoly

Forward Induction

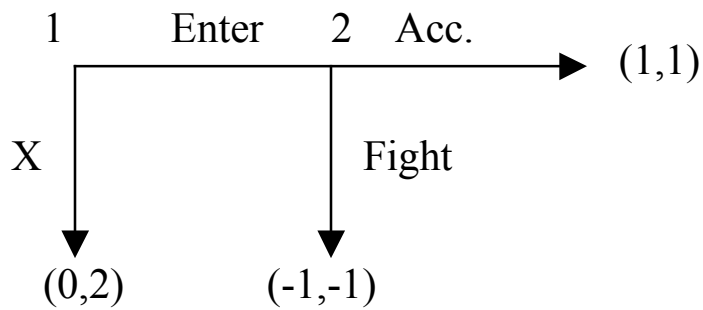
Strong belief in rationality: At any history of the game, each agent is assumed to be rational if possible. (That is, if there are two strategies s and s' of a player i that are consistent with a history of play, and if s is strictly dominated but s' is not, at this history no player j believes that i plays s .)

Burning Money

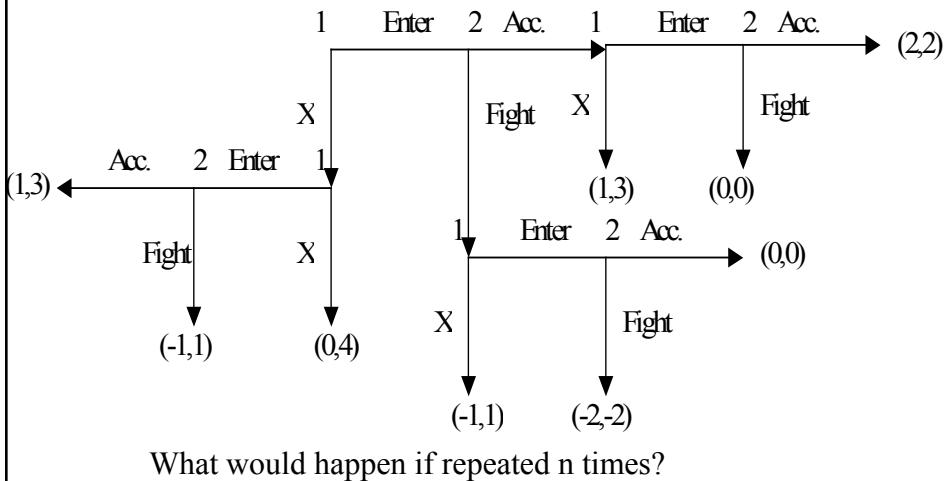


Repeated Games

Entry deterrence



Entry deterrence, repeated twice, many times

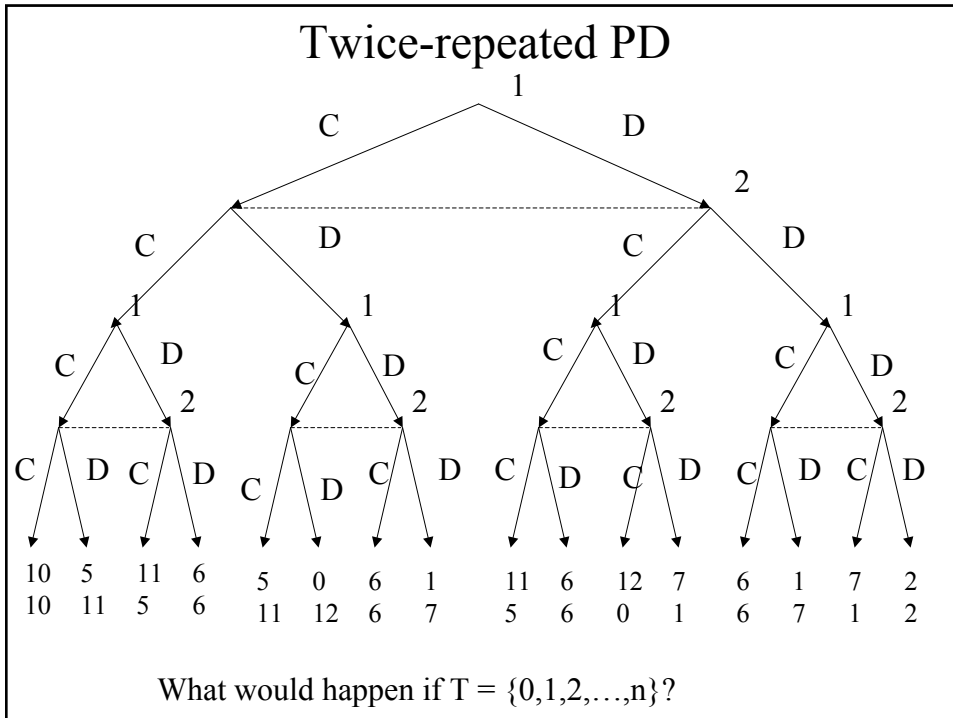


Prisoners' Dilemma, repeated twice, many times

- Two dates $T = \{0,1\}$;
- At each date the prisoners' dilemma is played:

	C	D
C	5,5	0,6
D	6,0	1,1

- At the beginning of 1 players observe the strategies at 0.
Payoffs= sum of stage payoffs.



A general result

- G = “stage game” = a finite game
- $T = \{0, 1, \dots, n\}$
- At each t in T , G is played, and players remember which actions taken before t ;
- Payoffs = Sum of payoffs in the stage game.
- Call this game $G(T)$.

Theorem: If G has a unique subgame-perfect equilibrium s^* , $G(T)$ has a unique subgame-perfect equilibrium, in which s^* is played at each stage.

With multiple equilibria

$$T = \{0,1\}$$

		2		
		L	M2	R
1	T	1,1	5,0	0,0
	M1	0,5	4,4	0,0
	B	0,0	0,0	3,3

Infinitely repeated Games with observable actions

- $T = \{0,1,2,\dots,t,\dots\}$
- $G =$ “stage game” = a finite game
- At each t in T , G is played, and players remember which actions taken before t ;
- Payoffs = Discounted sum of payoffs in the stage game.
- Call this game $G(T)$.

Definitions

The *Present Value* of a given payoff stream $\pi = (\pi_0, \pi_1, \dots, \pi_t, \dots)$ is

$$PV(\pi; \delta) = \sum_{t=1}^{\infty} \delta^t \pi_t = \pi_0 + \delta \pi_1 + \dots + \delta^t \pi_t + \dots$$

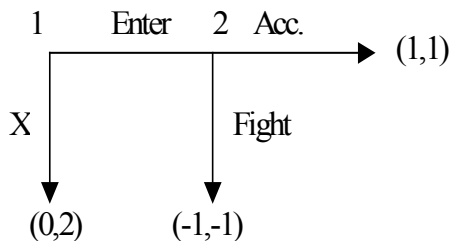
The *Average Value* of a given payoff stream π is

$$(1-\delta)PV(\pi; \delta) = (1-\delta) \sum_{t=1}^{\infty} \delta^t \pi_t$$

The *Present Value* of a given payoff stream π at t is

$$PV_t(\pi; \delta) = \sum_{s=t}^{\infty} \delta^{s-t} \pi_s = \pi_t + \delta \pi_{t+1} + \dots + \delta^s \pi_{t+s} + \dots$$

Infinite-period entry deterrence



Strategy of Entrant:

Enter iff
Accommodated before.

Strategy of Incumbent:

Accommodate iff
accommodated before.

Incumbent:

- $V(\text{Acc.}) = V_A =$
- $V(\text{Fight}) = V_F =$
- Case 1: Accommodated before.
 - Fight \Rightarrow
 - Acc. \Rightarrow
- Case 2: Not Accommodated
 - Fight \Rightarrow
 - Acc. \Rightarrow
 - Fight \Leftrightarrow

Entrant:

- Accommodated
 - Enter \Rightarrow
 - X \Rightarrow
- Not Acc.
 - Enter \Rightarrow
 - X \Rightarrow

Infinitely-repeated PD

	C	D
C	5,5	0,6
D	6,0	1,1

A Grimm Strategy:
Defect iff someone
defected before.

- $V_D = 1/(1-\delta)$;
- $V_C = 5/(1-\delta) = 5V_D$;
- Defected before (easy)
- Not defected
 - D \Rightarrow
 - C \Rightarrow
 - C \Leftrightarrow

Tit for Tat

- Start with C; thereafter, play what the other player played in the previous round.
- Is (Tit-for-tat, Tit-for-tat) a SPE?
- **Modified:** Start with C; if any player plays D when the previous play is (C,C), play D in the next period, then switch back to C.

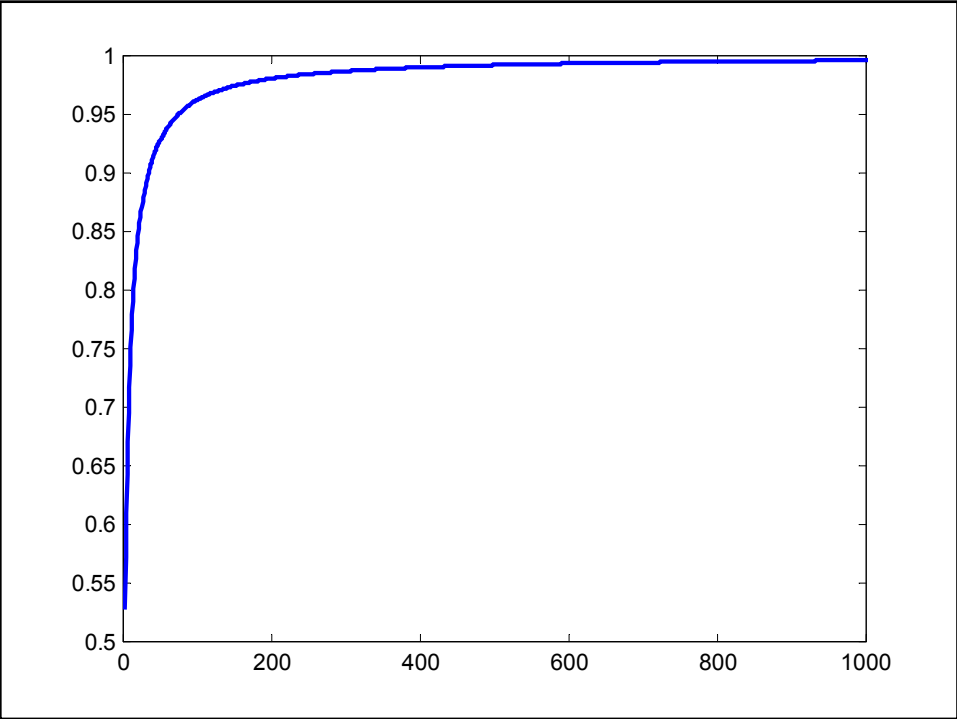
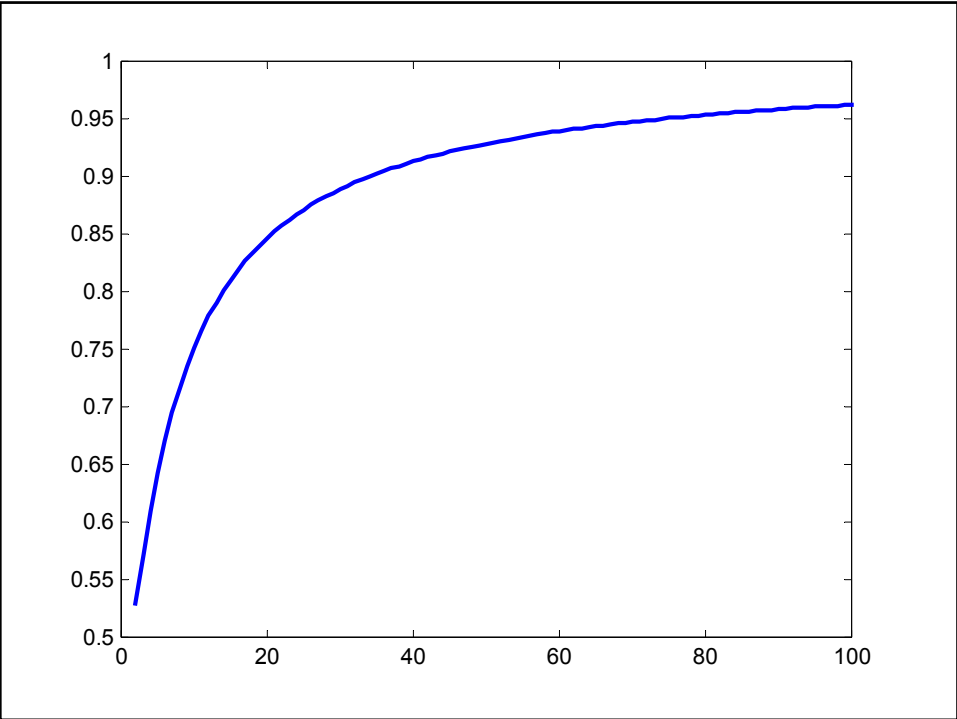
Folk Theorem

Definition: A payoff vector $v = (v_1, v_2, \dots, v_n)$ is feasible iff v is a convex combination of some pure-strategy payoff-vectors, i.e.,

$$v = p_1 u(a^1) + p_2 u(a^2) + \dots + p_k u(a^k),$$

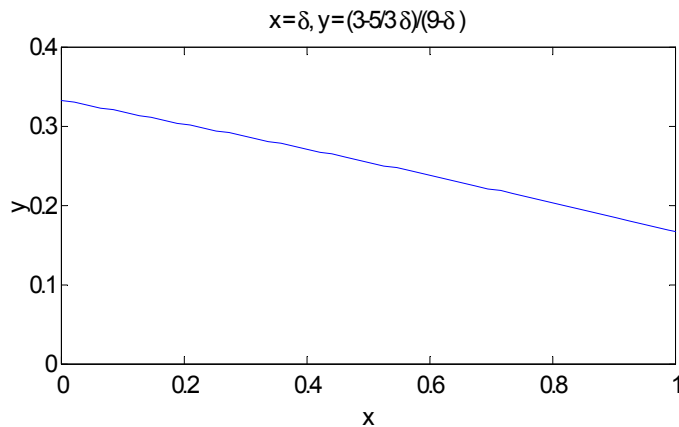
where $p_1 + p_2 + \dots + p_k = 1$, and $u(a^j)$ is the payoff vector at strategy profile a^j of the stage game.

Theorem: Let $x = (x_1, x_2, \dots, x_n)$ be a feasible payoff vector, and $e = (e_1, e_2, \dots, e_n)$ be a payoff vector at some equilibrium of the stage game such that $x_i > e_i$ for each i . Then, there exist $\underline{\delta} < 1$ and a strategy profile s such that s yields x as the expected average-payoff vector and is a SPE whenever $\delta > \underline{\delta}$.



IRCD (n=2)

- Strategy: Each firm is to produce q^* ; if any one deviates, each produce $1/(n+1)$ thereafter.
- $V_C = q^*(1-2q^*)/(1-\delta)$;
- $V_D = 1/(9(1-\delta))$;
- $V_{D|C} = \max_q q(1-q^*-q) + \delta V_D = (1-q^*)^2/4 + \frac{\delta}{9(1-\delta)}$
- Equilibrium iff
$$q^*(1-2q^*) \geq (1-\delta)(1-q^*)^2/4 + \delta/9$$
- $\Leftrightarrow q^* \geq \frac{9-5\delta}{3(9-\delta)}$



Carrot and Stick

Produce $\frac{1}{4}$ at the beginning; at ant $t > 0$, produce $\frac{1}{4}$ if both produced $\frac{1}{4}$ or both produced x at $t-1$; otherwise, produce x .

Two Phase: Cartel & Punishment

$$V_C = 1/8(1-\delta). \quad V_x = x(1-2x) + \delta V_C.$$

$$V_{D|C} = \max q(1-1/4-q) + \delta V_x = (3/8)^2 + \delta V_x$$

$$V_{D|x} = \max q(1-x-q) + \delta V_x = (1-x)^2/4 + \delta V_x$$

$$V_C \geq V_{D|C} \Leftrightarrow V_C \geq (3/8)^2 + \delta^2 V_C + \delta x(1-2x)$$

$$\Leftrightarrow (1-\delta^2) V_C - (3/8)^2 \geq \delta x(1-2x) \Leftrightarrow (1+\delta)/8 - (3/8)^2 \geq \delta x(1-2x)$$

$$V_x \geq V_{D|x} \Leftrightarrow (1-\delta)V_x \geq (1-x)^2/4 \Leftrightarrow (1-\delta)(x(1-2x) + \delta/8(1-\delta)) \geq (1-x)^2/4$$

$$\Leftrightarrow (1-\delta)x(1-2x) + \delta/8 \geq (1-x)^2/4$$

$$2x^2 - x + 1/8 - 9/64\delta \geq 0$$

$$(9/4-2\delta)x^2 - (3-2\delta)x + \delta/8(1-\delta) \leq 0$$