

Lectures 12-13
Incomplete Information
Static Case

14.12 Game Theory
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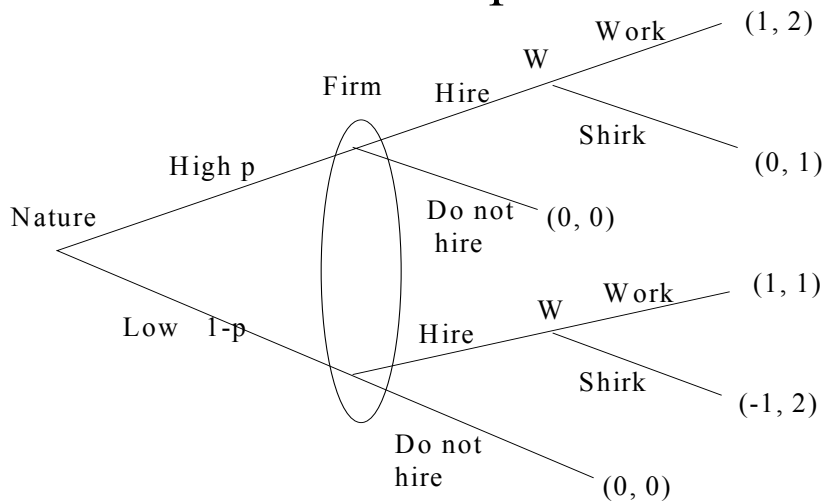
Road Map

1. Examples
2. Bayes' rule
3. Definitions
 1. Bayesian Game
 2. Bayesian Nash Equilibrium
4. Mixed strategies, revisited
5. Economic Applications
 1. Cournot Duopoly
 2. Auctions
 3. Double Auction

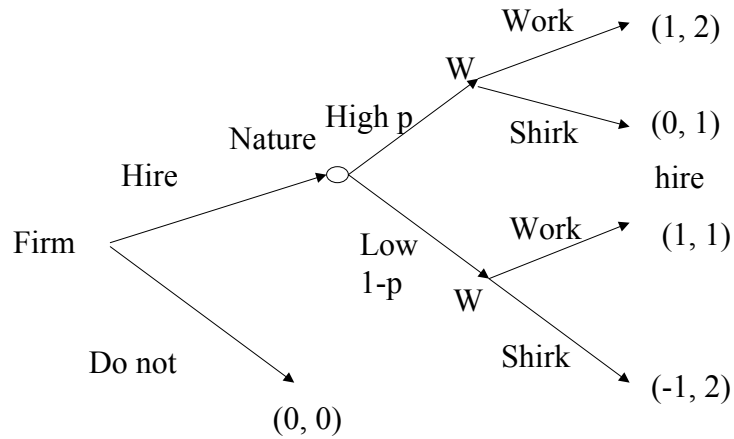
Incomplete information

We have incomplete (or asymmetric) information if one player knows something (relevant) that some other player does not know.

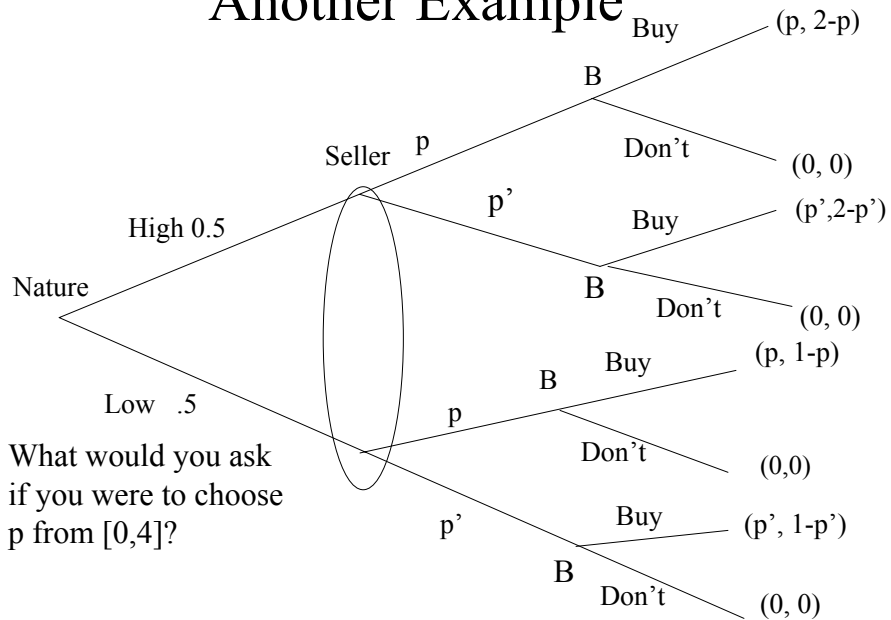
An Example



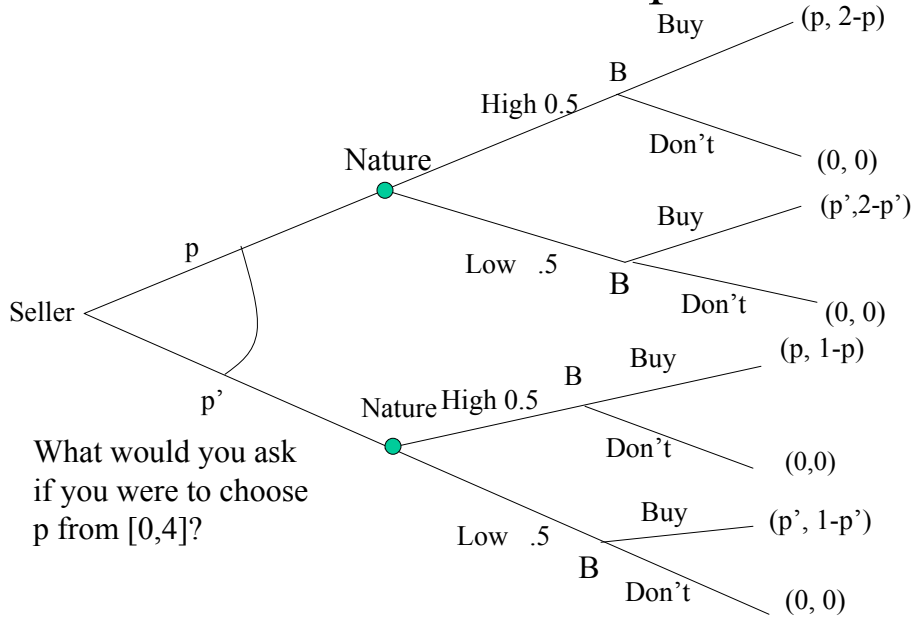
The same example



Another Example



Same “Another Example”



Bayes' Rule

Prob(A and B)

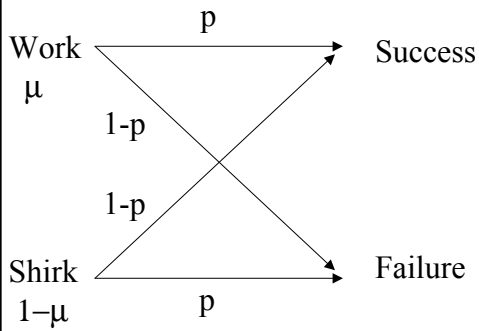
- $\text{Prob}(A|B) = \frac{\text{Prob(A and B)}}{\text{Prob(B)}}$

- $\text{Prob(A and B)} = \text{Prob}(A|B)\text{Prob}(B) = \text{Prob}(B|A)\text{Prob}(A)$

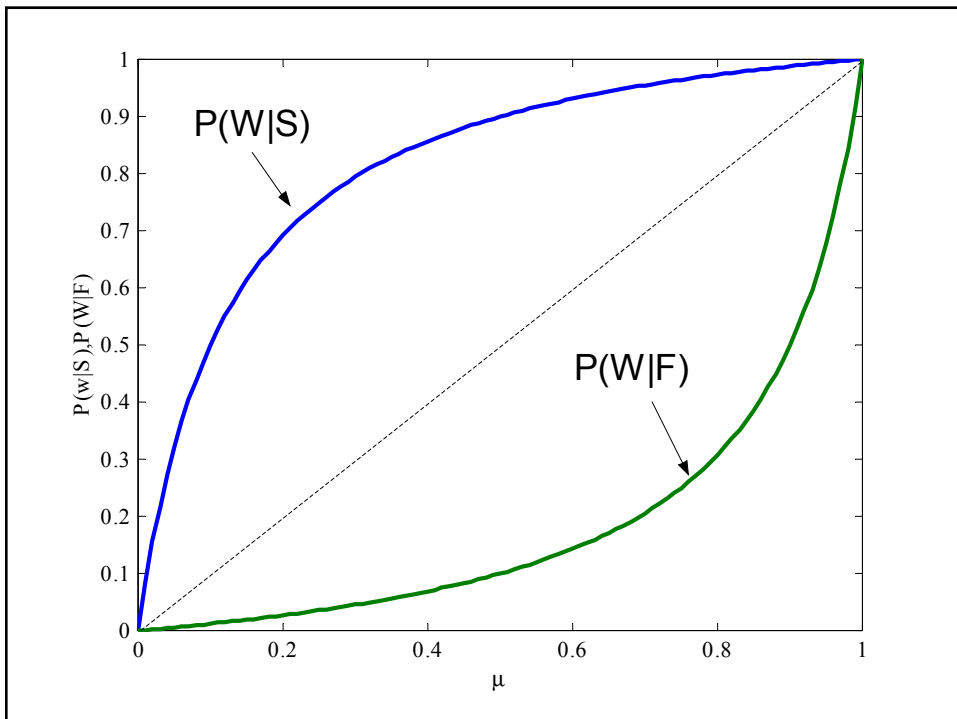
Prob(B|A)Prob(A)

- $\text{Prob}(A|B) = \frac{\text{Prob(B|A)Prob(A)}}{\text{Prob(B)}}$

Example



- $\text{Prob}(\text{Work}|\text{Success}) = \frac{\mu p}{\mu p + (1-\mu)(1-p)}$
- $\text{Prob}(\text{Work}|\text{Failure}) = \frac{(1-\mu)p}{\mu(1-p) + (1-\mu)p}$



Bayesian Game (Normal Form)

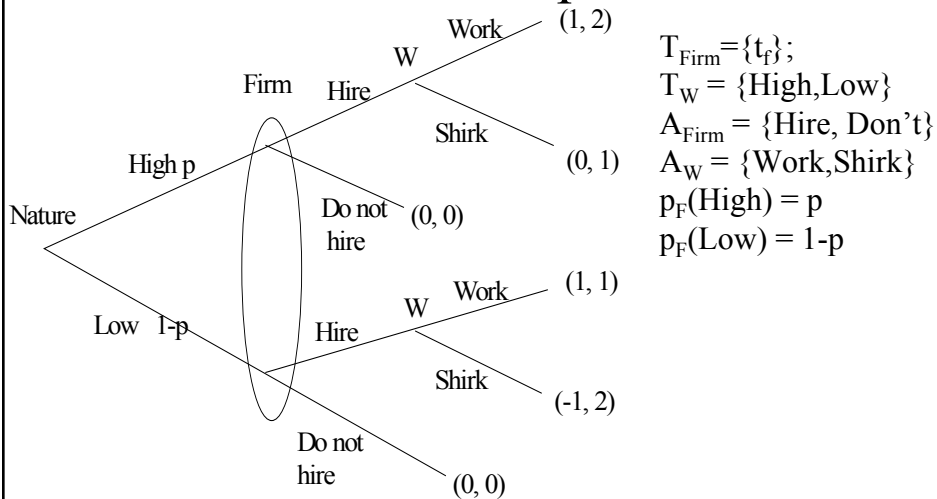
A Bayesian game is a list

$$G = \{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

where

- A_i is the action space of i (a_i in A_i)
- T_i is the type space of i (t_i)
- $p_i(t_{-i}|t_i)$ is i 's belief about the other players
- $u_i(a_1, \dots, a_n; t_1, \dots, t_n)$ is i 's payoff.

An Example



Bayesian Nash equilibrium

A Bayesian Nash equilibrium is a Nash equilibrium of a Bayesian game.

Given any Bayesian game $G =$

$$\{A_1, \dots, A_n; T_1, \dots, T_n; p_1, \dots, p_n; u_1, \dots, u_n\}$$

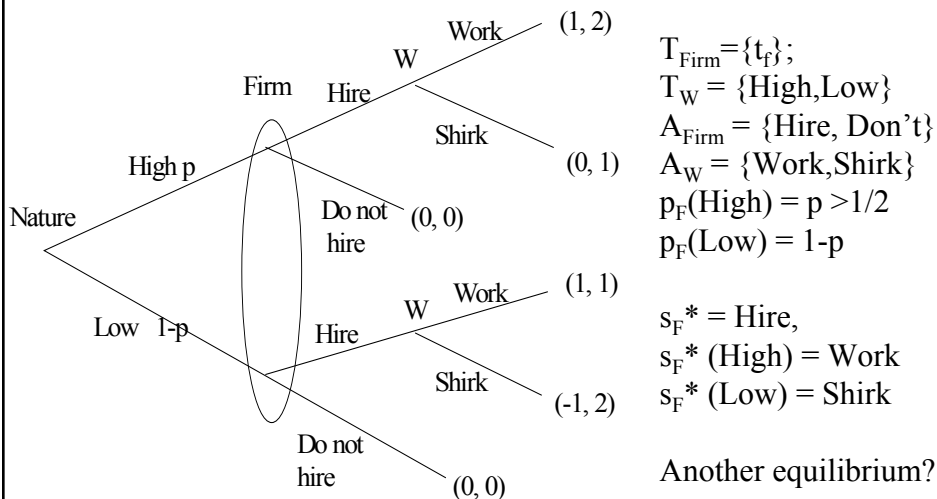
a **strategy** of a player i in a is any function $s_i: T_i \rightarrow A_i$;

A strategy profile $s^* = (s_1^*, \dots, s_n^*)$ is a **Bayesian Nash equilibrium** iff $s_i^*(t_i)$ solves





$$\max_{a_i \in A_i} \sum_{t_{-i} \in T_{-i}} u_i(s_1^*(t_1), \dots, s_{i-1}^*(t_{i-1}), a_i, s_{i+1}^*(t_{i+1}), \dots, s_n^*(t_n); t) p_i(t_{-i} | t_i)$$

i.e., s_i^* is a best response to s_{-i}^* .





An Example



Stag Hunt, Mixed Strategy

		
	(2,2)	(4,0)
	(0,4)	(6,6)

Mixed Strategies

		
	$2+t, 2+v$	$4+t, 0$
	$0, 4+v$	$6, 6$

- t and v are iid with uniform distribution on $[-\epsilon, \epsilon]$.
- t and v are privately known by 1 and 2, respectively, i.e., are types of 1 and 2, respectively.
- Pure strategy:
 - $s_1(t) = \text{Rabbit}$ iff $t > 0$;
 - $s_2(v) = \text{Rabbit}$ iff $v > 0$.
- $p = \text{Prob}(s_1(t) = \text{Rabbit} | v) = \text{Prob}(t > 0) = 1/2$.
- $q = \text{Prob}(s_2(v) = \text{Rabbit} | t) = 1/2$.

$$U_1(R|t) = t + 2q + 4(1-q) = t + 4 - 2q$$

$$U_1(S|t) = 6(1-q);$$

$$U_1(R|t) > U_1(S|t) \Leftrightarrow t + 4 - 2q > 6(1-q)$$

$$\Leftrightarrow t > 6 - 6q + 2q - 4 = 2 - 4q = 0.$$