Lectures 9

Single deviation-principle
&
Forward Induction

14.12 Game Theory

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Road Map

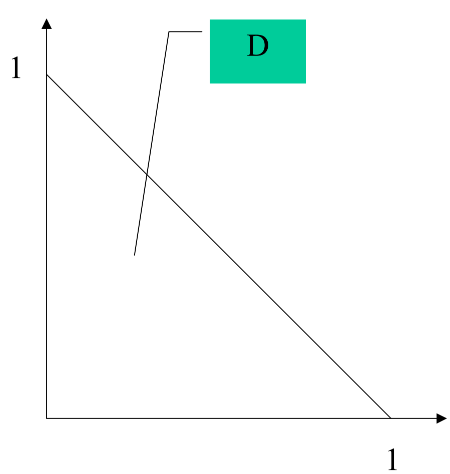
1. Single-deviation principle – Infinite-horizon bargaining
2. Quiz
3. Forward Induction – Examples
4. Finitely Repeated Games

Single-Deviation principle

Definition: An extensive-form game is *continuous at infinity* iff, given any $\varepsilon > 0$, there exists some t such that, for any two path whose first t acts are the same, the payoff difference of each player is less than ε.

Theorem: Let G be a game that is continuous at infinity. A strategy profile $s = (s_1, s_2, \ldots, s_n)$ is a subgame-perfect equilibrium of G iff, at any information set, where a player i moves, given the other players strategies and given i's moves at the other information sets, player i cannot increase his conditional payoff at the information set by deviating from his strategy at the information set.

Sequential Bargaining



- $N = \{1,2\}$
- D = feasible expected-utility pairs $(x,y \in D)$
- $U_i(x,t) = \delta_i^t x_i$
- $d = (0,0) \in D$ disagreement payoffs

Timeline – ∞ period

$T = \{1,2,\ldots, n-1,n,\ldots\}$

If t is odd,

- Player 1 offers some (x_t,y_t),
- Player 2 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

If t is even

- Player 2 offers some (x_t,y_t),
- Player 1 Accept or Rejects the offer
- If the offer is Accepted, the game ends yielding payoff $\delta^t(x_t,y_t)$,
- Otherwise, we proceed to date t+1.

SPE of ∞-period bargaining

Theorem: At any t, proposer offers the other player $\delta/(1+\delta)$, keeping himself $1/(1+\delta)$, while the other player accept an offer iff he gets $\delta/(1+\delta)$.

"Proof:"

Nash equilibria of bidding game

- 3 equilibria: s^1 = everybody plays 1; s^2 = everybody plays 2; s^3 = everybody plays 3.
- Assume each player trembles with probability $\varepsilon < 1/2$, and plays each unintended strategy w.p. $\varepsilon/2$, e.g., w.p. $\varepsilon/2$, he thinks that such other equilibrium is to be played.
  - s^3 is an equilibrium iff
  - s^2 is an equilibrium iff
  - s^1 is an equilibrium iff

Forward Induction

Strong belief in rationality: At any history of the game, each agent is assumed to be rational if possible. (That is, if there are two strategies s and s' of a player i that are consistent with a history of play, and if s is strictly dominated but s' is not, at this history no player j believes that i plays s.)

Bidding game with entry fee

Each player is first to decide whether to play the bidding game (E or X); if he plays, he is to pay a fee p > 60.

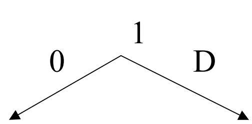
Bid \ min	1	2	3
1	60	-	-
2	40	80	-
3	20	60	100

For each m =1,2,3, ∃SPE: (m,m,m) is played in the bidding game, and players play the game iff $20(2+m) \geq p$.

Forward induction: when $20(2+m) < p$, (Em) is strictly dominated by (Xk). After E, no player will assign positive probability to min bid ≤ m. FI-Equilibria: (Em,Em,Em) where $20(2+m) \geq p$.

What if an auction before the bidding game?

Burning Money



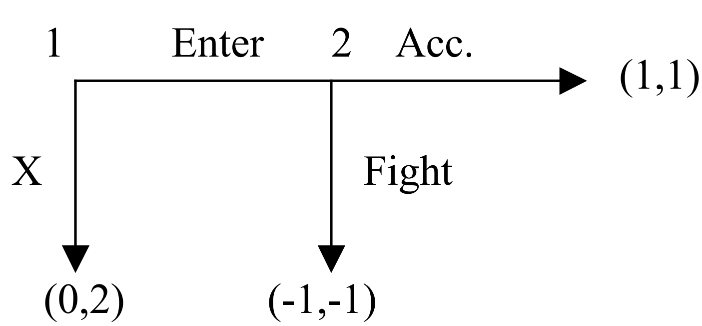
	B	S
B	3,1	0,0
S	0,0	1,3

	B	S
B	2,1	-1,0
S	-1,0	0,3

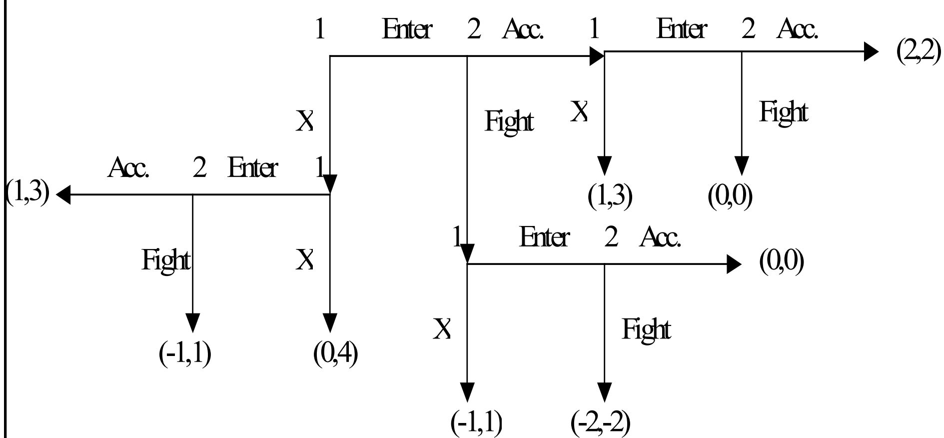
	BB	BS	SB	SS
0B				
0S				
DB				
DS				
O	T	H	E	R

Repeated Games

Entry deterrence



Entry deterrence, repeated twice



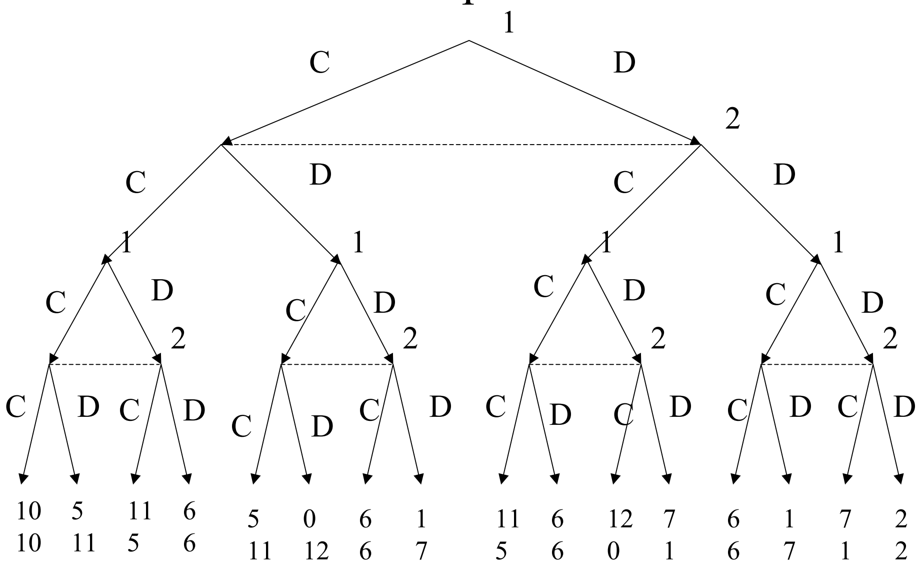
Prisoners' Dilemma, repeated twice, many times

- Two dates T = {0,1};
- At each date the prisoners' dilemma is played:

	C	D
C	5,5	0,6
D	6,0	1,1

- At the beginning of 1 players observe the strategies at 0. Payoffs= sum of stage payoffs.

Twice-repeated PD



What would happen if T = {0,1,2,…,n}?

A general result

- G = “stage game” = a finite game
- T = {0,1,…,n}
- At each t in T, G is played, and players remember which actions taken before t;
- Payoffs = Sum of payoffs in the stage game.
- Call this game G(T).

Theorem: If G has a unique subgame-perfect equilibrium s*, G(T) has a unique subgame-perfect equilibrium, in which s* is played at each stage.