

Answers to Midterm Exam

1. (i) normal form:

	<i>ll</i>	<i>lr</i>	<i>rl</i>	<i>rr</i>
<i>L</i>	(5, 5)	(5, 5)	(3, 3)	(3, 3)
<i>R</i>	(10, 0)	(-10, -10)	(10, 0)	(-10, -10)

(ii) NE are  $(R, ll)$  with payoffs (10, 0);  $(R, rl)$  with payoffs (10, 0); and  $(L, lr)$  with payoffs (5, 5)

(iii)  $(L, lr)$  is not reasonable because player 2 cannot credibly convince player 1 that he would play  $r$  in his lower node, since  $l$  is clearly better (0 is better than -10). Similarly  $(R, rl)$  is not convincing because player 2 clearly prefers  $l$  to  $r$  in his upper node.

(iv) SPE is  $(R, ll)$ . There is a subgame starting at each of player 2's nodes. Each one of these subgames has a unique NE which is  $l$  on the upper one and  $l$  on the lower one. This allows us to exclude  $r$  in both cases as a possible SPE outcome. Cutting off the "bad" branches we have

$$L \quad (5, 5)$$

$$R \quad (10, 0)$$

now the choice for player 1 is clear - he prefers  $R$ .

2. (i) Both firms have to choose wages above  $w_R$  and:

$$\Pi_1 = (y - w_1) \left( (1 - \lambda) \frac{N}{2} + \lambda N \right) \quad \text{if } w_1 > w_2$$

$$\Pi_1 = (y - w_1) \left( \frac{N}{2} \right) \quad \text{if } w_1 = w_2$$

$$\Pi_1 = (y - w_1) \left( (1 - \lambda) \frac{N}{2} \right) \quad \text{if } w_1 < w_2$$

and similarly for firm 2.

(ii) when  $\lambda = 0$

$$\Pi_1 = (y - w_1) \left( \frac{N}{2} \right) \quad \text{if } w_1 > w_2$$

$$\Pi_1 = (y - w_1) \left(\frac{N}{2}\right) \quad \text{if } w_1 = w_2$$

$$\Pi_1 = (y - w_1) \left(\frac{N}{2}\right) \quad \text{if } w_1 < w_2$$

That is, the profit does not change if a firm lowers the wage, so each firm will offer the lowest possible wage which is  $w_R$ .

(iii) when  $\lambda = 1$

$$\Pi_1 = (y - w_1) (N) \quad \text{if } w_1 > w_2$$

$$\Pi_1 = (y - w_1) \left(\frac{N}{2}\right) \quad \text{if } w_1 = w_2$$

$$\Pi_1 = (y - w_1) (0) \quad \text{if } w_1 < w_2$$

The firm with the lowest wage gets zero profit. No firm will be at equilibrium offering lower wage than the other. This means that wages offered are equal. But at any wage each firm wants to raise its wage just enough to be above the other's. So the only equilibrium is when neither wants to increase wage any more - that is -  $w_1 = w_2 = y$ .

(iv) when  $0 < \lambda < 1$ , rewriting profits

$$\Pi_1 = (y - w_1) \left(\frac{N}{2} + \lambda \frac{N}{2}\right) \quad \text{if } w_1 > w_2$$

$$\Pi_1 = (y - w_1) \left(\frac{N}{2}\right) \quad \text{if } w_1 = w_2$$

$$\Pi_1 = (y - w_1) \left(\frac{N}{2} - \lambda \frac{N}{2}\right) \quad \text{if } w_1 < w_2$$

As long as wages are below  $y$ , each firm prefers to increase wages rather than offering the same wage. If wages are equal to  $y$  (profits are zero), then firms prefer to lower wage all the way down to  $w_R$ . But both firms at this wage is also not an equilibrium, because each is willing to increase it a little.

3.(i) 100 is strictly dominated by 0, for both players. We can delete 100 for both. We cannot delete any more strategies. Note that, for example, 99, is not strictly dominated by 0 - if the other player says 100, 99 and 0 give exactly the same payoff. (99 is weakly dominated but not strictly dominated). Not strictly dominated are [0,99].

	0	1	2	. . .	98	99	100
0	5,5	10,0	10,0	. . .	10,0	10,0	10,0
1	0,10	5,5	10,0	. . .	10,0	10,0	10,0
2	0,10	0,10	5,5	. . .	10,0	10,0	10,0
.							
.							
.							
98	0,10	0,10	0,10		5,5	10,0	10,0
99	0,10	0,10	0,10		0,10	5,5	10,0
100	0,10	0,10	0,10		0,10	0,10	5,5

(ii) Now, once we have deleted 100 for both players, 99 is strictly dominated by 0. That's the only strategy we can eliminate in this iteration.

	0	1	2	. . .	98	99
0	5,5	10,0	10,0	. . .	10,0	10,0
1	0,10	5,5	10,0	. . .	10,0	10,0
2	0,10	0,10	5,5	. . .	10,0	10,0
.						
.						
.						
98	0,10	0,10	0,10		5,5	10,0
99	0,10	0,10	0,10		0,10	5,5

In each iteration we can eliminate the highest number. We are left with only [0] after 100 iterations.

(iii) The rationalizable solutions in a 2-player game are those that remain after strict dominance elimination - in this case it's only (0,0).

(iv) NE is (0,0) with payoffs (5,5).

(v) In an n-player game 0 by all players is a NE.