14.12 Economic Applications of Game Theory
Midterm Examination—October 16th, 1997

You have one and a half hours. Answer all three questions. The number of points for each part is given. They add up to 100.

1) Consider an individual with von Neumann-Morgenstern utility function $u(c) = \frac{c^{1+\alpha}}{1+\alpha}$. The individual has some wealth equal to W at the beginning of the period and does not consume until the end. He can either store his wealth, consumes it for certain at the end of the period, or he can invest part of it in the stock market. The individual knows that the stock market pays a gross rate of return $R > 1$ (that is every \$ invested becomes \$$R$) with probability p or collapses and pays nothing with probability $1 - p$.

1. Find the share of wealth x that the individual puts in the stock market. [9 points]

2. What happens to x as W increases? Why is this? [5 points]

3. What happens to x as α increases? Why is this? [5 points]

4. Now suppose that there is a financial analyst who can give advice. He either reports that the market will boom, in which case the return is R with probability $q > p$, or he reports that the market will crash in which case the return is R with probability $1 - q$. Explain why a rational investor should expect the analyst to report that the market will boom with probability p. [5 points]

5. Find the optimal investment strategy of the individual after listening to the advice of the analyst [Hint: you have to find two numbers x_g and x_b the respective proportions of wealth invested after a good and a bad report]. Explain carefully why the individual would pay money to listen to the analyst's advice. [7 points]

2) There are two risk-neutral workers and two jobs. One job offers wage w_1 and the other wage $w_2 < w_1$. Worker A first decides which job to apply to. Then Worker B observes worker A's choice and makes his decision. If there is one applicant for a job, he is employed and receives the associated wage. If both workers apply to the same job, then the firm randomly chooses between the two applicants, and the other is unemployed at wage 0.

1. Draw the extensive form of this game. [5 points]

2. Write down the normal form and find all the pure strategy equilibria. [If you find it useful, you can distinguish between two cases depending on whether $w_1 > 2 \times w_2$]. [10 points]

3. Now find the Subgame Perfect Equilibria. Explain carefully why some of the Nash Equilibria are not subgame perfect. [10 points]

4. Now assume that Worker B does not observe A's choice. Write down the extensive and the normal forms of the game, and find all Nash Equilibria (including mixed strategy equilibria). [9 points]

3) There are two coffee shops each with constant marginal cost of a cup of coffee equal to c. These two stores compete by setting prices. There are N consumers where N is a large number. Each consumer purchases one cup of coffee as long as the price is less than or equal to R. A fraction $1 - \lambda$ of these consumers see both prices and purchase at the lowest price (as long as $\leq R$). The remaining fraction λ randomly walk into one of the two stores and buy a cup as long as the price is less than or equal to R. If the price is greater than R, they do not buy any coffee.

1. Write down the profits of the two firms as functions of the two prices. [5 points]

2. Find the Nash Equilibria when $\lambda = 0$ and then when $\lambda = 1$. [8 points]

3. Show that there exists no pure strategy Nash Equilibria. [20 points]

4. Show that there exists no mixed strategy Nash Equilibria where one of the firms randomizes between a finite number of strategies. [Difficult. Only for bonus points. Do not waste time on this]