Midterm Exam 14.12

Answer all three questions. You have one and half hours. Each question is worth a 33 percent of the grade. Good luck.

1) Consider an individual with von Neumann-Morgenstern utility function $u(c) = \frac{1-\exp\{-\theta c\}}{\theta}$ with $\theta > 0$. The individual has some wealth equal to W at the beginning of the period and does not consume until the end. He can either store his wealth, consumes it for certain at the end of the period, or he can invest part of it in the stock market. The individual knows that the stock market pays a gross rate of return $R > 1$ (that is every \$ invested becomes \$$R$) with probability p or collapses and pays nothing with probability $1 - p$.

1. Find the amount I that the individual puts in the stock market.
2. What happens to I as W increases? What happens to the share of wealth invested? Why is this?
3. What happens to I as θ increases? Why is this?
4. Now suppose that there is a financial analyst who can give advice. He either reports that the market will boom, in which case the return is R with probability $q > p$, or he reports that the market will crash in which case the return is R with probability $1-q$. Find the optimal investment strategy of the individual after listening to the advice of the analyst [Hint: you have to find two numbers I_g and I_b the respective amounts of wealth invested after a good and a bad report]. Explain carefully why the individual would pay money to listen to the analyst's advice.
5. Discuss informally why the individual may not want to listen to the advice of the analyst if the analyst's pay is a fraction of the investment in the risky asset.

2) There are M coffee shops, each with cost of a cup of coffee equal to $c(x) = cx^2$ for x cups of coffee. These stores compete by setting prices. There are N consumers where N is a large number. Each consumer purchases one cup of coffee as long as the price is less than or equal to R. A fraction $1 - \lambda$ of these consumers see all prices and purchase at the lowest price (as long as $\leq R$). The remaining fraction λ randomly walk into one of the stores and buy a cup as long as the price is less than or equal to R. If the price is greater than R, they do not buy any coffee.

1. Write down the profits of each firm as a function of the prices of others.
2. Find the Nash Equilibria when $\lambda = 0$ and then when $\lambda = 1$.
3. Show that there exists no Nash Equilibria with all firms charging the same price when $\lambda \in (0, 1)$.

3) Consider the following two-period political game with two players, the working class and the elite. In the first-period, the elite decide whether to redistribute or not, and then the working class decides whether to carry out a revolution. Redistribution and no revolution gives a utility 10 to the working class and 15 to the elite. If there is no redistribution and no revolution, the working class gets 0 and the elite get 25. And if there is a revolution (irrespective of redistribution), the working class get 15 and the elite get 0. If there is a revolution, that is the end of the game, and the payoffs are final. If there is no revolution, the game proceeds to the second period, where both parties get additional payoffs. But first, nature determines whether the working class has the opportunity to carry out a revolution. The probability that this opportunity exists for the working class is q. Observing whether the working class has the opportunity to carry out a revolution, the elite again decide whether to redistribute. Once again, without redistribution, they get an additional 25 and the working class get 0. With redistribution, the working class get an additional 10 and the elite 15. Also, again a revolution gives 15 to the working class and nothing to the elite (irrespective of whether there is redistribution in the second period or not). There is no discounting between the two periods.

1. Draw the game tree.
2. Find the unique subgame perfect equilibrium of this game.
3. Explain why a high value of q, probability of revolution opportunity in the second period, prevents a revolution.