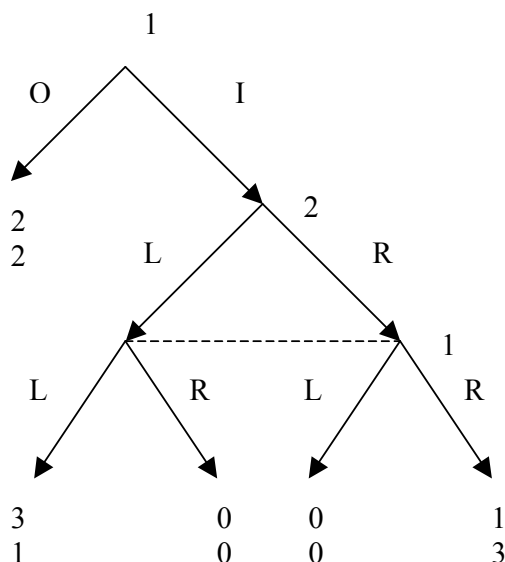**14.12 Game Theory – Final**
12/10/2001

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Instructions. This is an open book exam; you can use any written material. You have 2 hours 50 minutes. Each question is 20 points. Good luck!

1. Consider the following extensive form game.

(a) Find the normal form representation of this game.

(b) Find all rationalizable pure strategies.

(c) Find all pure strategy Nash equilibria.

(d) Which strategies are consistent with all of the following assumptions?

   (i) 1 is rational.
   (ii) 2 is sequentially rational.
   (iii) at the node she moves, 2 knows (i).
   (iv) 1 knows (ii) and (iii).

2. This question is about a milkman and a customer. At any day, with the given order,

   • Milkman puts $m \in [0,1]$ liter of milk and $1 - m$ liter of water in a container and closes the container, incurring cost cm for some $c > 0$;

   • Customer, without knowing m, decides on whether or not to buy the liquid at some price p. If she buys, her payoff is $vm - p$ and the milkman's payoff is $p - cm$. If she does not buy, she gets 0, and the milkman gets $-cm$. If she buys, then she learns m.

(a) Assume that this is repeated for 100 days, and each player tries to maximize the sum of his or her stage payoffs. Find all subgame-perfect equilibria of this game.

(b) Now assume that this is repeated infinitely many times and each player tries to maximize the discounted sum of his or her stage payoffs, where discount rate is $\delta \in (0,1)$. What is the range of prices p for which there exists a subgame perfect equilibrium such that, everyday, the milkman chooses $m = 1$, and the customer buys on the path of equilibrium play?

3. For the game in question 3.a, assume that with probability 0.001, milkman strongly believes that there is some entity who knows what the milkman does and will punish him severely on the day 101 for each day the milkman dilutes the milk (by choosing $m < 1$). Call this type irrational. Assume that this is common knowledge. For $v > p > c$, find a perfect Bayesian equilibrium of this game.

Bonus: [10 points] Discuss what would happen if the irrational type were known to dilute the milk by accident with some small but positive probability.

4. Find a perfect Bayesian equilibrium of the following game.
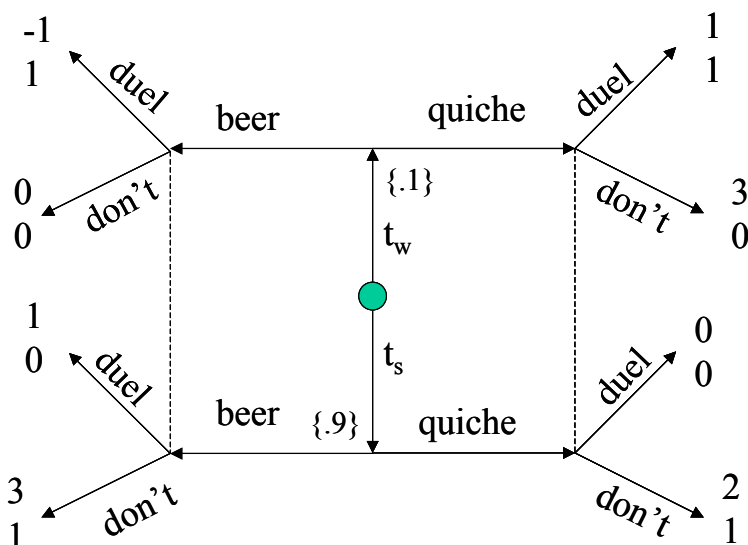
5. A risk-neutral entrepreneur has a project that requires $100,000 as an investment, and will yield $300,000 with probability 1/2, $0 with probability 1/2. There are two types of entrepreneurs: rich who has a wealth of $1,000,000, and poor who has $0. For some reason, the wealthy entrepreneur cannot use his wealth as an investment towards this project. There is also a bank that can lend money with interest rate π. That is, if the entrepreneur borrows $100,000 to invest, after the project is completed he will pay back $100,000 (1 + \pi)$ — if he has that much money. If his wealth is less than this amount at the end of the project, he will pay all he has. The order of the events is as follows:

- First, bank posts π.

- Then, entrepreneur decides whether to borrow ($100,000) and invest.
- Then, uncertainty is resolved.

(a) Compute the subgame perfect equilibrium for the case when the wealth is common knowledge.

(b) Now assume that the bank does not know the wealth of the entrepreneur. The probability that the entrepreneur is rich is $1/4$. Compute the perfect Bayesian equilibrium.