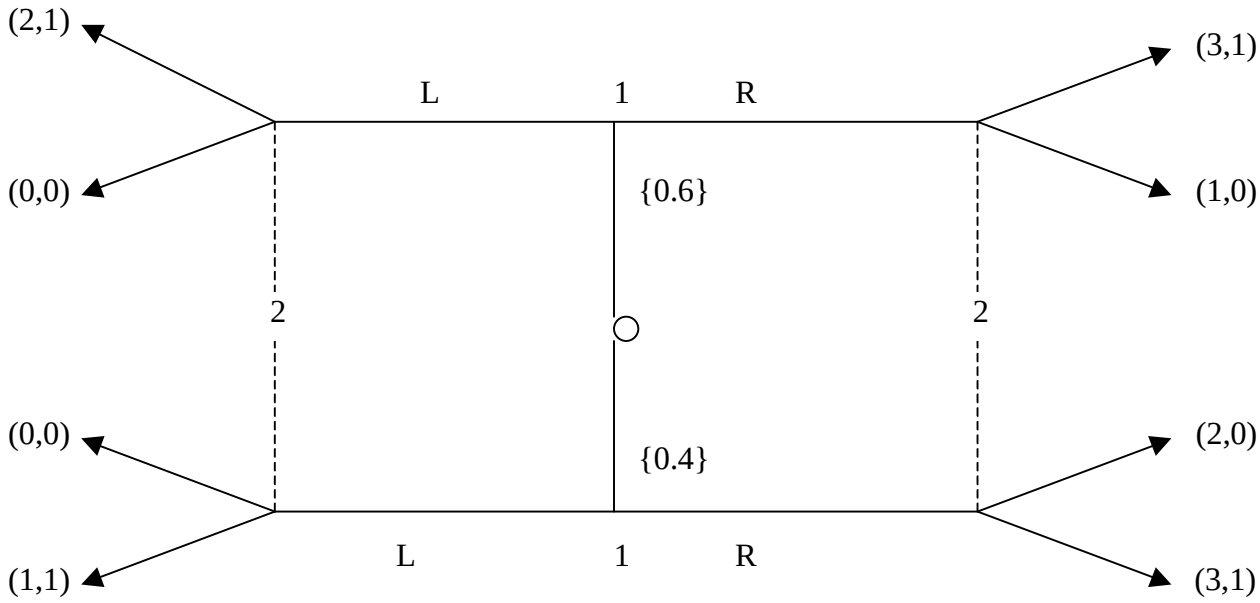

Figure 1:

14.12 Game Theory – Final (Make Up)
12/15/2000

Prof. Muhamet Yildiz
Instructions. This is an open book exam; you can use any written material. You have one hour and 20 minutes. Please answer only three of the following four questions. Each question is 33 points. Good luck!

1. Two players (say A and B) own a company, each of them owning a half of the Company. They want to dissolve the partnership in the following way. Player A sets a price p. Then, player B decides whether to buy A's share or to sell his own share to A,in each case at price p. The value of the Company for players A and B are v_A and v_B, respectively.

   (a) Assume that the values v_A and v_B are commonly known. What would be the price in the subgame-perfect equilibrium?

   (b) Assume that the value of the Company for each player is his own private information, and that these values are independently drawn from a uniform distribution on [0,1]. Compute the perfect Bayesian equilibrium.

2. Consider the following game.

   (a) Find a separating equilibrium.

   (b) Find a pooling equilibrium.

   (c) Find an equilibrium in which a type of player 1 plays a (completely) mixed strategy.

3. Find the subgame-perfect equilibrium of the following 2-person game. First, player 1 picks an integer x_0 with $1 \leq x_0 \leq 10$. Then, player 2 picks an integer y_1 with $x_0 + 1 \leq y_1 \leq x_0 + 10$. Then, player 1 picks an integer x_2 with $y_1 + 1 \leq x_2 \leq y_1 + 10$. In this fashion, they pick integers, alternatively. At each time, the player moves picks an integer, by adding an integer between 1 and 10 to the number picked by the other player last time. Whoever picks 100 wins the game and gets 100; the other loses the game and gets zero.

4. Consider a used-car market where buyers need to use their cars in two periods. There are two types of cars, peaches and lemons. A peach will not break down and the buyer will use it two periods. A lemon can break down at the end of the first period with probability 1/2, in which case buyer will not be able to use in the second period. The value of a car for a buyer is \$2,000 if he uses it only in the first period, and \$4,000 if he uses it in both periods. The value of a car for the seller is \$2,600 if it is a lemon, and \$3,600 if it is a peach. For each car the seller knows whether it is a lemon or peach but the buyer does not know. Both buyers and the sellers are risk neutral, and there are equal number of buyers and sellers (and equal number of peaches and lemons).

   (a) Find the equilibrium price in this market.

   (b) Now, introduce a risk-neutral dealer in the market. He can tell whether a car is a lemon or a peach with certainty. He sells the cars with a warranty, replacing the car with another one if it breaks down, incurring a cost of \$1,800. Now every seller either sells in the market or goes to the dealer. After checking the car, dealer offers him a price. If the seller accepts, the dealer buys the car at that price, and sells it in the market with the warranty. What are the equilibrium prices in the market? [The price of a car sold by the owner, by the dealer, and the amount the dealer offers to each seller.]