14.12 Game Theory
Prof. Muhamet Yildiz
TA: Kenichi Amaya
Fall 2001

Homework 1

Due on 9/26/2001 (in class)

1. Take $X = \mathbb{R}$, the set of real numbers, as the set of alternatives. Define a relation $\succeq$ on X by

$$x \succeq y \iff x \geq y - 1/2 \qquad \text{for all } x, y \in X.$$

   (a) Is $\succeq$ a preference relation? (Provide a proof.)

   (b) Define the relations $\succ$ and $\sim$ by

$$x \succ y \iff [x \succeq y \text{ and } y \not\succeq x]$$

   and

$$x \sim y \iff [x \succeq y \text{ and } y \succeq x],$$

   respectively. Is $\succ$ transitive? Is $\sim$ transitive? Prove your claims.

   (c) Would $\succeq$ be a preference relation if we had $X = \mathbb{N}$, where $\mathbb{N} = \{0, 1, 2, \ldots\}$ is the set of all natural numbers?

2. We have two dates: 0 and 1. We have a security that pays a single dividend, at date 1. The dividend may be either \$100, or \$50, or \$0, each with probability 1/3. Finally, we have a risk-neutral agent with a lot of money. (The agent will learn the amount of the dividend at the beginning of date 1.)

   (a) An agent is asked to decide whether to buy the security or not at date 0. If he decides to buy, he needs to pay for the security only at date 1 (not immediately at date 0). What is the highest price π_S at which the risk-neutral agent is willing to buy this security?

   (b) Now consider an "option" that gives the holder the right (but not obligation) to buy this security at a strike price K at date 1 — after the agent learns the amount of the dividend. If the agent buys this option, what would be the agent's utility as a function of the amount of the dividend?

   (c) An agent is asked to decide whether to buy this option or not at date 0. If he decides to buy, he needs to pay for the option only at date 1 (not immediately at date 0). What is the highest price π_O at which the risk-neutral agent is willing to buy this option?

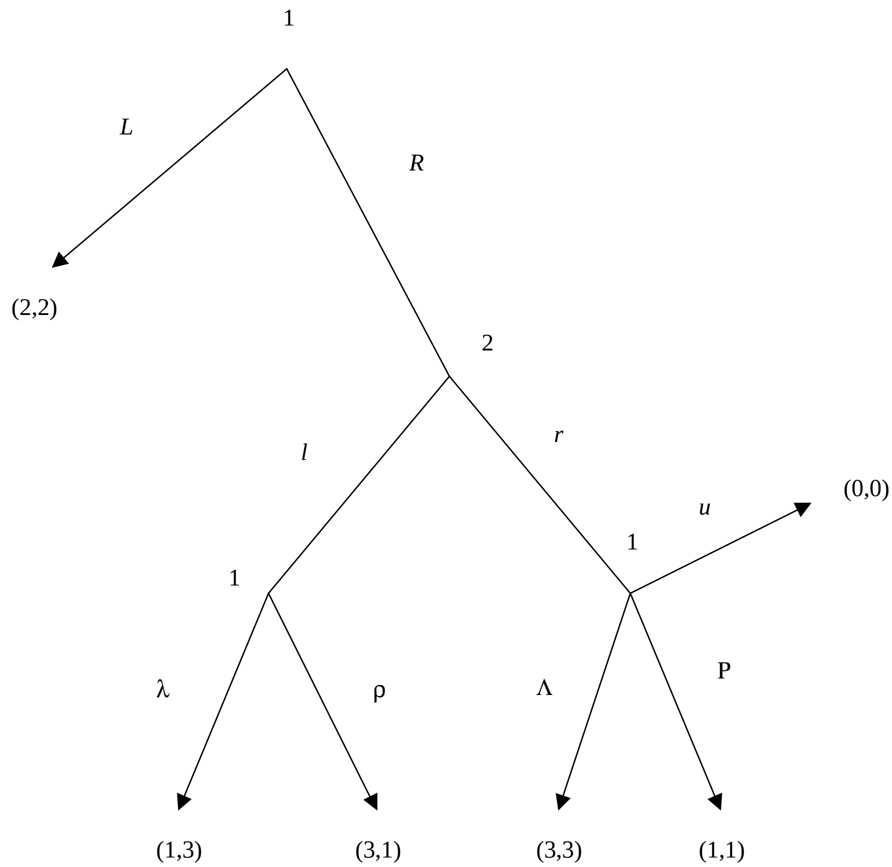
3. Construct a 2-player game with the following property. Player 1 has strategies s, s', and s'' such that neither s nor s' strictly dominates s'', but the mixed strategy that assigns probability 1/2 to each of the strategies s and s' strictly dominates s''.

4. Compute the set of rationalizable strategies in the following game that is played in a class of n students where $n \geq 2$: Without discussing with anyone, each student i is to write down a real number $x_i \in [0, 100]$ on a paper and submit it to the TA. The TA will then compute the average

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}$$

of these numbers. The students who submit the number that is closest to $\bar{x}/3$ will share the total payoff of 100, while the other students get 0. Everything described above is common knowledge. (Bonus: would the answer change if the students did not know n, but it were common knowledge that $n \geq 2$?)

5. Consider the following game in extensive form.



(a) Write this game in the strategic form.

(b) What are the strategies that survive the *iterative elimination of weakly-dominated strategies* in the following order: first eliminate all weakly-dominated strategies of player 1; then, eliminate all the strategies of player 2 that are weakly dominated in the remaining game; then, eliminate all the strategies of player 1 that are weakly dominated in the remaining game, and so on?