Economic Applications of Game Theory. Second Midterm Exam. November 20th.

You have one and a half hours. Answer all three questions. [Note: the questions do not have equal weight].

1) [Total: 30 points] Consider the following alternating offer bargaining game between two players A and B. A has a discount factor δ_A and B has δ_B. Initially, they have a cake of size equal to 2. In the first period, A makes an offer. B can either accept or reject. If he rejects this offer, then B gets to make an offer in the second period. A can accept or reject. If A rejects, however, in the third period, the cake shrinks to size 1 (this shrinkage is additional to the usual discounting between periods). From then on, the cake remains of size 1 (but there is still discounting) and the two players alternate in making offers until an offer is accepted.

Draw the game tree and find the unique subgame perfect equilibrium [Hint: first solve the subgame which starts in period 3 using standard techniques, and then use backward induction to find the equilibrium behavior in the first two periods.]

2) [Total: 40 points] Consider the following stage game between players A and B:

$A \setminus B$	l	m	r
L	$(-10, 4)$	$(10, 0)$	$(-1, -1)$
M	$(0, 10)$	$(-1, -1)$	$(-1, 1)$
R	$(4, -10)$	$(-1, -1)$	$(2, 2)$

1. Find the Nash Equilibria of this game. [5 points]
2. Consider a supergame G^T which is obtained by repeating this stage game T times. Find the subgame perfect equilibria of G^T. [5 points]
3. Now consider $G^\infty(\delta)$ which is obtained by repeating this stage game an infinite number of times with discount factor δ for both players. Find an equilibrium which is preferred to playing the Nash Equilibrium of the stage game. [30 points]

3)[Total: 30 points] Consider the following incomplete information Bertrand game. There is one customer who will buy one unit of the good as long as the price, P, is less than R. Firm 1 has cost equal to c. Firm 2 has cost c_L with probability θ and cost c_H with probability $1-\theta$, where $c_L < c < c_H < R$. The cost level of firm 2 is not observed by firm 1, but firm 2 knows its own cost level. Both firms simultaneously choose their prices. The firm with the lower price supplies the customer. If they have equal price, one of the firms is randomly chosen to supply the customer. Both firms are risk-neutral.

1. Write the payoff functions of the two firms. [5 points]
2. Define a Bayesian Nash Equilibrium for this game. [5 points]
3. Show that there exists no pure strategy Bayesian Nash Equilibrium.[20 points]