Economic Applications of Game Theory: 14.12

Mock Midterm

1. Consider the infinite horizion bargaining game. John and Beth are trying to split a pie of size 1. They both get linear utility from the share of the cake they obtain (x_J for John and x_B for Beth). Both players discount every period of bargaining with discount factor δ (thus the whole cake next period is equivalent to a share δ of the cake this period). Also, each player can stop bargaining at any point and get his or her outside option d_J and d_B where $d_J + d_B < 1$ (the outside options are also discounted, thus receiving outside option d next period is worth δd this period).

   At the beginning of every period Nature decides which player will make the offer in that period. Both players have a probability $\frac{1}{2}$ of being selected every period (draws independent over time).

   (a) Sketch the game tree.

   (b) Find the subgame perfect equilibrium assuming that $d_J = d_B = 0$.

   (c) How does the equilibrium change when $d_J > 0$ and $d_B > 0$.

   (d) Compare this to the Nash Solution.

2. Consider the following Prisoner's Dilemma type game:

	Cooperate	Cheat
Cooperate	10,10	-1,11
Cheat	11,-1	0,0

Suppose that this game is repeated over time, and we are trying to maintain (Cooperate, Cooperate) as a subgame perfect equilibrium. Also assume that both players have discount factor equal to $\frac{1}{2}$.

(a) Find the trigger strategies that will support (Cooperate, Cooperate) as an equilibrium.

(b) Next, suppose that 'non-forgiving' strategies are not allowed. Instead consider the following Trigger strategy: "If you cheat I will cheat for the next T periods, and then I will cooperate again until you cheat one more time". Write the payoff to cheating in the first period, and then starting to cooperate in period T+1. Show that if T is greater than a cut-off level T^*, then cooperating is preferred to cheating.

(c) Explain in words why it is OK to look at the strategy of cheating now and then cooperating from T+1 onwards rather than cheating all the time?