

Economic Applications of Game Theory: 14.12

Second Midterm

Please Answer all three questions. You have 1.30 hours.

1. Consider the infinite horizon bargaining game. John and Beth are trying to split a pie of size 1. They both get linear utility from the share of the cake they obtain (x_J for John and x_B for Beth). Both players discount every period of bargaining with discount factor δ (thus the whole cake next period is equivalent to a share δ of the cake this period). Also, each player can stop bargaining at any point and get his or her outside option d_J and d_B where $d_J + d_B < 1$ (the outside options are also discounted, thus receiving outside option d next period is worth δd this period).

At the beginning of every period Nature decides which player will make the offer in that period. Both players have a probability $\frac{1}{2}$ of being selected every period (draws independent over time).

- (a) Sketch the game tree.
 - (b) Find the subgame perfect equilibrium assuming that $d_J = d_B = 0$.
 - (c) How does the equilibrium change when $d_J > 0$ and $d_B > 0$.
2. Consider the following Prisoner's Dilemma type game:

	Cooperate	Cheat
Cooperate	10,10	-1,11
Cheat	11,-1	0,0

Suppose that this game is repeated over time, and we are trying to maintain (Cooperate, Cooperate) as a subgame perfect equilibrium. Also assume that both players have discount factor equal to $\frac{1}{2}$.

- (a) Find the trigger strategies that will support (Cooperate, Cooperate) as an equilibrium.
 - (b) Next, suppose that ‘non-forgiving’ strategies are not allowed. Instead consider the following Trigger strategy: ”If you cheat I will cheat for the next T periods, and then I will cooperate again until you cheat one more time”. Write the payoff to cheating in the first period, and then starting to cooperate in period T+1. Show that if T is greater than a cut-off level T^* , then cooperating is preferred to cheating.
 - (c) Explain in words why it is OK to look at the strategy of cheating now and then cooperating from T+1 onwards rather than cheating all the time?
3. Consider the following incomplete information Bertrand game. Each player can be high or low type with probability $\frac{1}{2}$. These draws are independent. A high type has marginal cost 1 and a low type has marginal cost 0. There

is one unit of demand in the market for all prices less than $R(> 1)$. Thus, the profit of a firm who charges price $p < R$ while the other firm charges a price $p' > p$ is equal to $p - c$ where c is the relevant marginal cost.

- (a) Show that there exists no pure strategy Bayesian Nash Equilibrium.
- (b) **(Difficult- so do not get stuck on this part at the expense of the rest) Let us find the mixed strategy equilibrium. First show that the high types will set $p = 1$. Suppose that player 1 plays the mixed strategy $f(p)$ when he is of low type, where $f(p)$ is the probability density with which he chooses price p . Let $F(p)$ be the cdf. Explain why in this case the profit of the player 2 who chooses price q when he is low type is given as $\pi(q) = \frac{1}{2}q + \frac{1}{2}q[1 - F(q)]$ (where this is the ex ante profit before knowing the type of the other player). Then note that for a player of type low to play a mixed strategy, he needs to be indifferent between different prices. Using this information, solve for $F(p)$ such that player 2 is indifferent between different q 's. Using the information that $F(1) = 1$ characterize $F(p)$ completely. Then explain in words why both players playing $F(p)$ when they are low type and $p = 1$ when they are high type is a Bayesian Nash Equilibrium.**