Mock Solutions

(1) (a)

(b) Here a symmetric SPE is John (or Beth) always demands a share $1 - \frac{6}{7}$ if he (she) has the offer and accepts any share greater than or equal to \(\frac{6}{7}\) if he (she) doesn’t have the offer. If say John had the offer, he knows Beth won’t take any offers smaller than her discounted expected payoff if she waits a period. Since we are looking for symmetric equilibria here, this means she’ll accept only offers greater than or equal to \(x_B\) and he’ll only make offers to her less than or equal to \(x_B\) where \(x_B = \delta(\frac{1}{7}x_B + \frac{1}{7}(1 - x_B))\). Solving for \(x_B\) we find \(x_B = \frac{6}{7}\).

(c) If \(d_J\) and \(d_B\) are both positive, then it changes the outcome as follows: if both are less than \(\frac{6}{7}\) then things are unchanged. If say \(d_J > \frac{6}{7}\) and \(d_B \leq \frac{6}{7}\) then he demands shares of \(1 - \frac{6}{7}\), and she demands shares of \(1 - d_J\) respectively if he (she) has the offer. John accepts only offers greater than or equal to \(d_J\) and Beth only accepts offers greater than or equal to \(\frac{6}{7}\).

(2) (a) Clearly the following strategy will work - I start by playing cooperate. As long as the other player has always played cooperate, I continue to cooperate. If the other player ever cheats, then I play cheat forever. Here the subgames are just the game from any period \(t\) on. Thus I can consider any subgame starting when one player has defected when everyone cooperated until that point and consider the payoffs then. Suppose a player is considering deviating at period \(t\). The payoff to cooperating is \(10 + \frac{1}{2}10 + \frac{1}{2}10 + \ldots = \frac{10}{2} = 20\) while the payoff to cheating is period \(t\) is \(11 + 0 + 0 + 0 + \ldots = 11\). Cooperation has a higher payoff so it’s sustained.
(b) Now I will cooperate if the payoff from cheating and then cooperating once the punishment is over is less than that from cooperating the entire time. Since cheating only entails deviating from the cooperation strategy for the next $T + 1$ periods, and I will be back on the equilibrium path then, I only need to worry about the difference in the payoffs for the next $T + 1$ periods. If I cheat I get
\[ 11 + 0 + 0 + \ldots + 0 + 20 \left( \frac{T}{2} \right) \]
while if I don’t cheat I get 20. I will continue to cooperate only if the payoff from not cheating is bigger than that from cheating or if
\[ 2T \geq \frac{20}{9}. \]

Clearly this happens for all $T \geq T_\ast$ where $T_\ast = 1$. $T = 0$ does not work - we need some punishment to sustain cooperation.

(c) Because we are trying to enforce a non-cheating equilibrium we can suppose that in the future there will be no cheating, and only need to check the conditions necessary for preventing cheating this period.