Fall 2003

14.126 (Game Theory)
Problem Set 1

1. This exercise asks you to compare the equilibria of two games. Game A is a game of perfect information: Player 1 moves first, choosing U or D. Player 2 sees player 1's move and chooses L or R. The payoff matrix is

	L	R
U	(2,1)	(0,0) .
D	(4,–1)	(1,1)

In game B, player 1 again moves first, choosing U or D. Player 2 sees a noisy signal of 1's play: there are two possible values of the signal, s' and s'', with $\Pr(s'|U) = \Pr(s''|D) = 1-\varepsilon$. Player 2 then chooses either L or R. The payoff functions are the same as before, and in particular depend only on the actions chosen and not on the signal. For a non-empty interval of ε's that includes 0, characterize the set of (all) Nash equilibria of game B, and compare the resulting payoffs to the payoffs of the Nash and subgame-perfect equilibria of game A. Relate this to the upper and lower hemi-continuity of the correspondences mapping ε to the sets of Nash and SPE payoffs.

2. Prove that $K_i(E) = K_i\left(K_i(E)\right)$ and $\neg K_i(\neg K_i(E)) \subseteq K_i(E)$ (where "$\neg$" means "not".)

3. FT 14.1

4. This exercise looks at modeling types with possibility maps instead of probabilistic beliefs. Given a set X, let N(X) denote the set of all nonempty subsets of X. Fix a nonempty set S that is meant to model payoff-relevant uncertainty, as in Brandenburger-Dekel. An S-based possibility structure is a structure (S,T_1,T_2,ν_1,ν_2) where T_1 and T_2 are non-empty sets of "types" for players 1 and 2 respectively, and ν_i is a map from T_i to $N(S\times T_j)$. The set $\nu_i(t_i)$ is the possibility set of type t_i of player i- the set of , (s,t_j) pairs that t_i thinks are possible.

The possibility structure is called *complete* if the maps ν_1 and ν_2 are onto.
In this case every possibility set that is possible is actually present- there are no "holes."

Show that if the S has more than one element, there is no complete S-based possibility structure. (*Hint:* Show that when S has more than one element, there is no onto map from S to $N(S)$.)

Discuss. How does the assumption of completeness relate to the role of the measurable structure in Brandenburger and Dekel 1993?