Fall 2003

Economics 14.126
Problem Set 2
Due Wednesday Oct 29

1. Verify the claim in the proof of part (i) of Lemma 3.1 in Fudenberg-Levine (JET 94.)

2. FT 5.10.

3. Consider the following version of a repeated prisoner's dilemma. In the stage game, players choose simultaneously between *C* and D; the expected payoffs to an action profile are given by

	C	D
C	$1,1$	$-h,1+d$,
D	$1+d,-h$	$0,0$

where *d* and *h* are positive, and $d-h<1$.

There are two public outcomes, x',x''. The probability distribution over these outcomes is symmetric: $\pi_x(C,D)=\pi_x(D,C)$, and satisfies $\pi_{x'}(C,C)<\pi_{x'}(C,D)$ and $\pi_{x'}(C,D)>\pi_{x'}(D,D)$.

a) Verify that the expected payoff given above are consistent with a model where each player's realized payoff depends only on her own action and the public outcome.

b) Show that the set $\lim_{\delta\to 1} E(\delta)$ is bounded away from the efficient frontier, where $E(\delta)$ is the set of payoff vectors of perfect public equilibria.

c) For a given function π, let $\gamma=\pi_{x'}(C,D)-\pi_{x'}(C,C)$ and let $\beta=\pi_{x'}(C,D)-\pi_{x'}(D,D)$. Consider a sequence of games indexed by *n* that all have the expected payoff function given above, where both γ_n and β_n converge to 0, and the ratio $\frac{\gamma_n}{\gamma_n+\beta_n}$ converges to 0 as well. What is $\lim_{n\to\infty}\lim_{\delta\to 1} E(\delta)$?

d) Discuss this example. How do your findings relate to the fact that no action profile is pairwise identifiable?

4. Consider a repeated game of adverse selection, of the type described in Section 8 of FLM [94], where the player's stage-game is a report of their type, but suppose that the distribution *p* over types is not a product measure. Suppose that each player *i* has only 2 possible types, and that *p* puts positive probability on each of the possible type profiles.

a) Does the game have a product structure?

b) Suppose there are only two players. Is truthful reporting pairwise identifiable for generic distributions*?*

c) Suppose there are 3 players. Find a profile that is not pairwise identifiable.

d) Still with three players, show that the truthful reporting profile is pairwise identifiable for generic probability distributions p.

5) (extra credit) This question asks you to extend the characterization of $\lim_{\delta \to 1} E(\delta)$ to cases where the set $Q = \bigcap_{\lambda} H(\lambda)$ is lower dimensional. To do this, think of iterating the definition of *Q*, where at the first round is the original definition, and at each subsequent round *k* the set of current-period payoffs $g(\alpha)$ is constrained to lie in the set Q_{k-1} that was obtained in the previous round, and the continuation payoffs w are constrained to be lie in the affine hull of Q_{k-1} (that is, each *w* must be a linear combination of payoff vectors in Q_{k-1}, but the weights need neither be non-negative nor to sum to 1.) Show that this iterative process is well defined, and that the limit set Q^* equals $\lim_{\delta \to 1} E(\delta)$.