14.126 Game Theory Problem Set 3

Due on 11/24/2003

1. Consider the following Hawk-Dove game, where doves also incur some small cost t when they meet each other:

	H	D
H	$(v-w)/2, (v-w)/2$	$v, 0$
D	$0, v$	$v/2-t, v/2-t$

.

   (a) Assuming that $w > v$ and $t > 0$ find all evolutionarily stable strategies.

   (b) Now consider a third type, B, who plays H if he is the first to come to a territory and plays D (without incurring the cost t) if he is the second:

	H	D	B
H	$(v-w)/2, (v-w)/2$	$v, 0$	$3v/4 - w/4, (v-w)/4$
D	$0, v$	$v/2-t, v/2-t$	$v/4 - t/2, 3v/4 - t/2$
B	$(v-w)/4, 3v/4 - w/4$	$3v/4 - t/2, v/4 - t/2$	$v/2, v/2$

.

   For which values of v, w, and t, B is an evolutionarily stable strategy?

2. Consider the following "ultimatum game," where Player 1 offers some $a \in A = \{0.01, 0.02, \ldots, 1\}$, and Player 2 demands some $b \in A$. If $a \geq b$ (i.e., if Player 2 accepts the offer a), then the payoffs are $(1-a, a)$; otherwise, the payoffs are $(0, 0)$.

   (a) Compute all Nash equilibria.

   (b) Now consider an evolutionary process in which the members of a population are matched in pairs to play this ultimatum game where each agent is equally likely to play the roles of Player 1 and Player 2. Assume that the growth of the strategies in this role-playing game follows the replicator dynamics. Find all asymptotically stable strategies.

3. Consider a linear Cournot duopoly with the true inverse demand function $P = a - Q$ and zero marginal costs, where P is price, $a > 0$, and $Q = q_1 + q_2$ is the total supply of a good. Now imagine that each firm $i \in N = \{1, 2\}$ perceives the inverse demand function as $P = a + b_i - Q$, where $b_i \in \mathbb{R}$ is the bias in i's perception. Let $G(b_1, b_2)$ be the game game in which b_1 and b_2 are common-knowledge.

(a) Show that $G(b_1, b_2)$ has a unique rationalizable strategy profile. Compute the true payoffs $u_1(b_1, b_2)$ and $u_2(b_1, b_2)$ at the rationalizable strategy profile — computed by using $P = a - Q$.

(b) Consider the meta game $\Gamma = (N, \mathbb{R}, \mathbb{R}, u_1, u_2)$, where the strategies are choices of b_1 and b_2, and u_1 and u_2 are as in (a). Show that Γ has a unique Nash equilibrium b^*, and that $b_i^* > 0$ for each $i \in N$. Show that the replicator dynamics for Γ (using the true payoffs) converges to b^*.

(c) Now consider an evolutionary learning process in which the agents not only develop their perceptions (i.e., b_1 and b_2) but also learn how to play the game $G(b_1, b_2)$ given perceptions. Assume that the learning process is a "two-tiered" replicator dynamics in which they learn how to play $G(b_1, b_2)$ infinitely faster than they change their perceptions, i.e., given any perception-pair (b_1, b_2), the play converges to the limit of the dynamics for fixed (b_1, b_2) before they change their perceptions. What is the limit of this "two-tiered" replicator dynamics?

(d) Briefly discuss these results.

[**Hint:** Throughout this exercise, assume that the result of Samuelson and Zhang (i.e., Proposition 3.4 in FL or Theorem 3.1 in Weibull) applies.]

4. Consider the symmetric 2×2 coordination game with payoff matrix

	A	B
A	$1, 1$	$0, l$
B	$l, 0$	$1/2, 1/2$

where $l \in (1/2, 1)$ so that (B, B) is risk-dominant. This game is played by a single population of N agents. Each period, an agent is selected at random from the population, and given the chance to revise her strategy. With probability ε she switches to the other strategy, and with probability $1 - \varepsilon$ she decides what to do by observing both (a) the realization of a random "aspiration level" that is drawn from the uniform distribution on $[0, 1]$ and (b) the current strategy of one of the N players (including her own as one of the N for simplicity.) In this case, she switches strategy if and only if the payoff of her current strategy when matched with the sampled strategy is less than the aspiration level.

(Note that this is an example of what is called a "birth-death" process, as the state can change by at most 1 in each period. There is a well developed theory for such processes that you might want to investigate.)

What can you say about the limit of the ergodic distributions as $\varepsilon \to 0$ for fixed N? What about the limit as $N \to \infty$ for a fixed sufficiently small ε? Hint: the risk-dominant equilibrium is not always selected.