14.160 Experimental Economics
Problem Set 1 Solutions

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1 Fehr-Schmidt and Redistribution

1.1) The median voter has wage $w = \frac{W}{2}$ and thus has after-tax income of

$$\begin{aligned} y_{W/2} &= (1-t)\frac{W}{2} + (t-\gamma t^2)\frac{W}{2} \\ y_{W/2} &= \frac{W}{2}(1-\gamma t^2) \end{aligned}$$

Since linear utility was specified the median voter solves

$$\max_t u(\alpha y + \beta) = \max_t y_{W/2}$$

which yields $t^* = 0$.

1.2) Given linear utility, the central planner solves the following problem:

$$\max_t \bar{y}$$

Since y is a linear function of w and w is uniformly distributed, the mean will be the same as the median and the maximization problem will be the same as in part 1, and thus $t^* = 0$.

1.3) Now we compute an agent's utility using Fehr-Schmidt preferences:
First define $\bar{y}$ and $\underline{y}$, the maximum and minimum incomes, respectively.

$$\bar{y} = \frac{W}{2}(2 - t - \gamma t^2)$$

$$\underline{y} = \frac{W}{2}(t - \gamma t^2)$$

Since w is uniform over [0,w] and y is a linear transformation of w, y will be uniform over $\left[\underline{y}, \bar{y}\right]$.

$$u(y_w) = y_w - \alpha \int_{y_w}^{\bar{y}} (y - y_w) \frac{1}{W(1-t)} dy - \beta \int_{\underline{y}}^{y_w} (y_w - y) \frac{1}{W(1-t)} dy$$

The integration is straightforward and we find

$$u(y_w) = y_w - \frac{\alpha}{2W(1-t)}(\bar{y} - y_w)^2 - \frac{\beta}{2W(1-t)}(\underline{y} - y_w)^2$$

where, as before

$$y_w = (1-t)w + (t - \gamma t^2)W/2$$

$$\text{Equivalently, } \ u(y_w) = y_w + \frac{t-1}{2W}\left[\alpha(W-w)^2 + \beta w^2\right]$$

1.4) To solve for median voter optimum we set $w = W/2$ and maximize over t. First let's simplify

$$u(y_{W/2}) = y_{W/2} + \frac{1}{8}W(t-1)(\alpha+\beta)$$

Now we differentiate and solve:

$$\begin{aligned}
\frac{\partial u(y_{W/2})}{\partial t} &= -\gamma t W + \frac{1}{8}W(\alpha+\beta) = 0 \\
t^* &= \frac{\alpha+\beta}{8\gamma}
\end{aligned}$$

Note that in the limit as α and β go to zero we have standard linear utility, and in fact we get $t = 0$, the same as we found above.

Also, the optimal choice of t is invariant with respect to W. This makes sense because the individuals care about relative, not absolute wealth.

1.5) The central planner solves the following problem:

$$\max_t \int_0^1 u(y_w)dF(y_w)$$

$$\max_t \int_{\underline{y}}^{\bar{y}} \left[y_w - \frac{\alpha}{2W(1-t)}(\bar{y} - y_w)^2 - \frac{\beta}{2W(1-t)}(\underline{y} - y_w)^2\right] \frac{1}{W(1-t)} dy_w$$

The integration yields

$$\max_t \frac{W}{2}\left\{1 - \gamma t^2 - \left[\frac{(\alpha+\beta)(1-t)}{3}\right]\right\}$$

$$\begin{aligned}
2\gamma t &= \frac{\alpha+\beta}{3} \\
t^* &= \frac{\alpha+\beta}{6\gamma}
\end{aligned}$$

The social planner chooses a higher rate of taxation than the median worker. This is to be expected because the social planner is concerned with wokers throughout the wage distribution, who on net prefer more redistribution than the median.

2 Fehr-Schmidt and Contracts

a & b) We can solve for the optimal trust contract by using backward induction. The selfish principal wants to maximize $M^p = 10e - w$, so he offers $w = 0$. At stage 2, a selfish agent always chooses $e = 1$. A fair agent chooses e such that $M^p = 10e - w = w - c(e) = M^A$ This equation implicitly defines the fair agent's effort response function $e(w)$. By the implicit function theorem we have:

$$\begin{aligned} 10\frac{de}{dw} &= 2 - c'(e)\frac{de}{dw} \\ \frac{de}{dw} &= \frac{2}{10 + c'(e)} \end{aligned}$$

Consider now stage 1 and let q be the fraction of fair agents. Then, the expected monetary payoff of the principal from offering w is:

$$M^P(w) = q[10e(w) - w] + (1 - q)[10 - w]$$

Differentiating with respect to w yields:

$$\frac{\partial M^P}{\partial w} = 10q\frac{de}{dw} - 1 = \frac{20q}{10 + c'(e)} - 1 \leq 0 \text{ for } q < 0.55$$

Thus, there are two candidates for the optimal wage offer:

• $w = 0$ will be accepted by a selfish agent who then chooses $e = 1$ which yields a monetary payoff of 10 to the principal and 0 to the agent. However, the fair agent will reject this wage offer, so that both parties get a payoff of 0. If $q = 0.4$, the expected monetary payoff of the principal from offering $w = 0$ is thus $M^P = 6$.

• $w = 5$ will be accepted by both types of agents who both choose $e = 1$ yielding monetary payoffs $M^P = M^A = 5$. Hence, the selfish principal chooses $w = 0$ while the fair principal chooses $w = 5$. The selfish agent chooses $e = 1$ in response, since it minimizes cost of effort, while the fair agent chooses $e = 1$ because it equalizes payoffs, $M^P = M^A = 5$.

c & d) Consider the contract $(w = 4, e = 4, f = 13)$ which is optimal if all agents are selfish. If this contract is accepted and if the agent chooses $e = 4$ then monetary payoffs are given by $M^P = 26$ and $M^A = 0$. However, a fair agent will reject this contract. If $q = 0.4$, then the expected monetary payoff to the principal from this contract is $M^P = (1 - q)26 = 15.6$. In order to get a fair agent to accept the contract and choose $e = 4$, the principal would have to offer at least $w = 17$ which gives monetary payoffs $M^P = M^A = 13$ to both parties. Note that this is less for the principal than the "selfish offer" considered above. The principal could raise w above 17 in order to induce the fair agents to choose a higher effort level. However, by the same argument that was used for the trust contract, this does not pay off in expectation if $q < 0.6$.

e) When given a choice between trust and incentive contracts, all principals will choose an incentive contract because they yield higher expected payoffs, regardless of type. The expected payoffs to the principal and agent are as follows:

$$M^P = 0.6 * 15.6 + 0.4 * 13 = 14.56$$

$$M^A = 0.6 * 0 + 0.4 * 13 = 5.2$$

f) Suppose there is such a separating equilibrium. If the principal offers $\underline{w}$, the agent updates his beliefs to $\hat{q} = 0$. Thus, the agent is sure that the bonus is not going to be paid, so both types of agents will choose $e = 1$. Hence, the selfish principal's equilibrium payoff is $M^{P_S} = 10 - \underline{w} < 10$. Note that $\underline{w} = 0$ is the only possible equilibrium candidate. On the other hand, the selfish principal can always mimic the fair principal by choosing $w = \bar{w}$. In this case the agent believes $\hat{q} = 1$. We have to distinguish two cases:

- If $\bar{w} \leq 40$, both types of agent choose $e = 10$. Thus, if the selfish principal offers $w = \bar{w}$ and does not pay the bonus, her monetary payoff is $M^{P_S} = 100 - \bar{w} > 60$, a contradiction to the assumption that $w = \underline{w}$ is an equilibrium wage offer.

- If $\bar{w} > 40$, the selfish type of agent will choose $e = 1$ and the fair type of agent will choose $e = 10$. If the fraction of fair agents is 0.4, this gives an expected monetary payoff to the selfish principal who does not pay the bonus of $M^{P_S} = 0.4 \cdot 100 + 0.6 \cdot 10 - \bar{w} = 46 - \bar{w} < 10$. Hence, the selfish principal would have no incentive to deviate from $w = \underline{w} = 0$. However, in this case the fair principal wants to deviate. She pays the bonus to the agent who chooses $e = 10$, so her expected monetary payoff is $M^{P_R} = 0.4 \cdot 40 + 0.6 \cdot (10 - \bar{w}) < -2$. In addition, the fair principal suffers from the inequality if the selfish agent chooses $e = 1$. On the other hand, she could have guaranteed herself a payoff of 5 by offering $w = 5$ which would have been accepted by both agents, both agents would have chosen $e = 1$, and both parties would have had a payoff of 5. Hence, $\bar{w} > 40$ cannot be part of a separating equilibrium either.

g) At stage 3, the selfish principal always chooses $b = 0$. The fair principal chooses b in order to achieve

$$M^P = 10e - w - b = w + b - c(e) = M^A$$

Thus,

$$b(e) = max\{5e - w + \frac{c(e)}{2}, 0\}$$

At stage 2 all agents belief that they face the fair principal with probability 0.4 (because we are in a pooling equilibrium). Thus the agent's expected monetary payoff as a function of e is

$$M^A(e) = 0.4[w + b(e) - c(e)] + 0.6[w - c(e)]$$

Differentiating with respect to e yields:

$$\frac{\partial M^A}{\partial e} = \begin{cases} 0.4(5 + 0.5c'(e)) - c'(e) = 2 - 0.8c'(e) & \text{if } b(e) > 0 \\ -c'(e) & \text{if } b(e) = 0 \end{cases}$$

Thus, $M^A(e)$ has a local maximum at $e = 1$ and, given the convex cost function, another local maximum at $e = 7$. It is easy to check that the global maximum is at $e = 7$ for $w \leq 15$ and $e = 1$ for $w > 15$. Hence, the selfish agent chooses

$$e(w) = \begin{cases} 7 \text{ if } w \leq 15 \\ 1 \text{ if } w > 15 \end{cases}$$

A fair agent suffers from the inequality if the principal does not pay the bonus. Therefore, a fair agent rejects if $w < 5$ and accepts and chooses $e(w)$ such that $10e - w = w - c(e)$, which is equal to 1 if $5 \leq w \leq 10$ and equal to 2 if $10 < w \leq 15$.

Consider now stage 1: The fair principal would never offer $w > 15$. If $w > 15$, only the fair agents would choose $e > 1$. For the same argument as the one given in Proposition 1, increasing wages in order to appeal to the fair agents does not pay off in expected terms and the fair principal would suffer from the inequality to her disadvantage if the selfish agent chooses $e = 1$. Therefore, $w > 15$ cannot be part of a pooling equilibrium.