

14.20 Solutions to Midterm

Problem #1

- (a) Yes. An oligopolist will always produce on the elastic portion of its *residual* demand curve and may produce on the elastic portion of the *industry* demand curve.
- (b) When Southwest entered, it took Northwest's most price sensitive consumers, leaving Northwest with fewer consumers, but these consumers had less elastic demands. (This is a shift and a rotation of Northwest's demand). Because their consumers had less elastic demands, Northwest could *raise* the price it charged these buyers. Recall that p is inversely related to the elasticity of demand through the Lerner Index.
- (c) Any thoughtful story that made sense and made some reference to the readings was given credit.
- (d) False. This is the Demsetz critique. The appearance of a positive relationship between market concentration and industry profits could be driven by cost asymmetries across firms. If firms are not equally efficient (i.e.: some firms have lower costs than others), then the more efficient firms will have larger market shares and will appear to have higher profits. This will show up as a positive correlation between concentration and profits. However, the higher profitability in the more concentrated industry is due to the cost advantages of the larger firms and *not* due to market power.

Problem #2

- (a) strategy:

$$(i) \quad \underbrace{s_{i,t}(h^{t-1})}_{\substack{\text{strategy of} \\ \text{firm } i \text{ + as a} \\ \text{function of the} \\ \text{history of prices} \\ \text{up to period } (t-1)}} = \begin{cases} p^M & \text{if } p_j^{t-1} = p^M \forall j \quad \text{or} \\ h_j^{t-1} = \underbrace{(c, c, \dots, c)}_{T \text{ periods}} \forall j \\ c & \text{otherwise} \end{cases}$$

(ii) Payoff from following: $\frac{\pi^M}{N(1-\delta)}$

Payoff from deviating: $\pi^M + 0(\delta + \delta^2 + \dots + \delta^T) + \frac{\pi^M}{N}\delta^{T+1} + \dots$

∴ Need:

$$\begin{aligned}
\frac{\pi^M}{N(1-\delta)} &\geq \pi^M + \frac{\pi^M}{N} \left(\frac{\delta^{T+1}}{1-\delta} \right) \\
&\Rightarrow \frac{1}{N(1-\delta)} \geq 1 + \frac{\delta^{T+1}}{N(1-\delta)} \\
&\Rightarrow 1 \geq N(1-\delta) + \delta^{T+1} \\
&\Rightarrow \frac{1-\delta^{T+1}}{1-\delta} \geq N
\end{aligned}$$

$\uparrow N \Rightarrow$ Harder to sustain collusion; δ must \uparrow

$\uparrow T \Rightarrow$ Easier to sustain collusion

(b) Now observe rival's price with a $K - pd$ lag:

(only consider first $(K + T)$ pds)

Payoff from following: $\frac{\pi^M}{N}(1 + \delta + \delta^2 + \dots + \delta^{K+T})$

Payoff from deviating: $\pi^M(1 + \delta + \delta^2 + \dots + \delta^K) + 0(\delta^{K+1} + \dots + \delta^{K+T})$

\therefore Need:

$$\begin{aligned}
\frac{\pi^M}{N}(1 + \delta + \dots + \delta^{K+T}) &\geq \pi^M(1 + \delta + \dots + \delta^K) \\
&\Rightarrow (1 + \delta + \dots + \delta^{K+T}) \geq N(1 + \delta + \dots + \delta^K) \\
\frac{1 - \delta^{K+T+1}}{1 - \delta} &\geq N \left(\frac{1 - \delta^{K+1}}{1 - \delta} \right) \\
&\Rightarrow \frac{1 - \delta^{K+T+1}}{1 - \delta^{K+1}} \geq N
\end{aligned}$$

$\uparrow K$ makes it harder to sustain collusion because monopoly profits can be obtained for a longer period of time.

(c) Let π^H be the monopoly profits in the *high* state.

Let π^L be the monopoly profits in the *low* state.

(i) Suppose demand is *HIGH* today:

Follow:

$$\begin{aligned}
&\frac{\pi^H}{2} + \delta \left[\frac{1}{2} \left(\frac{\pi^H}{2} \right) + \frac{1}{2} \left(\frac{\pi^L}{2} \right) \right] + \delta^2 [\cdot] + \dots \\
&= \frac{\pi^H}{2} + \frac{\delta}{1-\delta} \left[\frac{\pi^H + \pi^L}{4} \right]
\end{aligned}$$

Deviate: $\pi^H + \delta \cdot 0 + \dots$

\therefore Need:

$$\begin{aligned}\frac{\pi^H}{2} + \frac{\delta}{1-\delta} \left[\frac{\pi^H + \pi^L}{4} \right] &\geq \pi^H \\ \Rightarrow \frac{\delta}{1-\delta} \left[\frac{\pi^H + \pi^L}{4} \right] &\geq \frac{\pi^H}{2} \\ \Rightarrow \frac{\delta}{1-\delta} [\pi^H + \pi^L] &\geq 2\pi^H \\ \Rightarrow \frac{\delta}{1-\delta} &\geq \frac{2\pi^H}{\pi^H + \pi^L}\end{aligned}$$

$$\delta = \frac{1}{2}$$

$$\text{LHS: } \frac{\delta}{1-\delta} = 1$$

$$\text{RHS: } \frac{2\pi^H}{\pi^H + \pi^L} = 1 \text{ only if } \pi^H = \pi^L$$

\therefore When $\delta = 1/2$, can only sustain π^L in high demand state.

$$\delta = 2/3$$

$$\text{LHS: } \frac{\delta}{1-\delta} = 2$$

$$\text{RHS: } \frac{2\pi^H}{\pi^H + \pi^L} < 2 \text{ as long as } \pi^L > 0, \text{ which we can assume.}$$

\therefore When $\delta \geq 2/3$, we can sustain π^H in high demand state.

\Rightarrow For all $1/2 < \delta < 2/3$, in order to sustain collusion in the high demand state, the collusive price (in the high state) must be lowered below p^{MH} in order to give profits below π^H .

(ii) *Suppose demand is LOW today*

Then need:

$$\begin{aligned}
\frac{\pi^L}{2} + \frac{\delta}{1-\delta} \left[\frac{\pi^L + \pi^H}{4} \right] &\geq \pi^L \\
\frac{\delta}{1-\delta} \left[\frac{\pi^L + \pi^H}{4} \right] &\geq \frac{\pi^L}{2} \\
\hline
\frac{\delta}{1-\delta} (\pi^L + \pi^H) &\geq 2\pi^L \\
\frac{\delta}{1-\delta} &\geq \frac{2\pi^L}{\pi^L + \pi^H} \\
\delta &= 1/2
\end{aligned}$$

$$\text{LHS: } \frac{\delta}{1-\delta} = 1$$

$$\text{RHS: } \frac{2\pi^L}{\pi^L + \pi^H} < 1 \text{ since } \pi^L < \pi^H$$

\therefore Collusion sustainable for $\delta \geq \frac{1}{2}$

\Rightarrow Collusion sustainable for $\frac{1}{2} < \delta < \frac{2}{3}$

Problem #3

$$(a) \pi_1(q_1, q_2) = (40 - q_1 - q_2 - c_1)q_1$$

FOC:

$$\begin{aligned}
40 - q_1 - q_2 - c_1 - q_1 &= 0 \\
2q_1 &= 40 - q_2 - c_1 \\
q_1 &= \frac{40 - q_2 - c_1}{2}
\end{aligned}$$

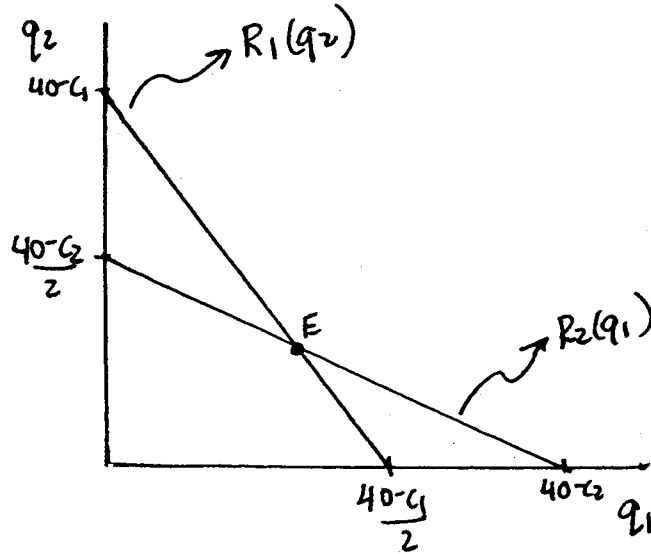
By symmetry:

$$q_2 = \frac{40 - q_1 - c_2}{2}$$

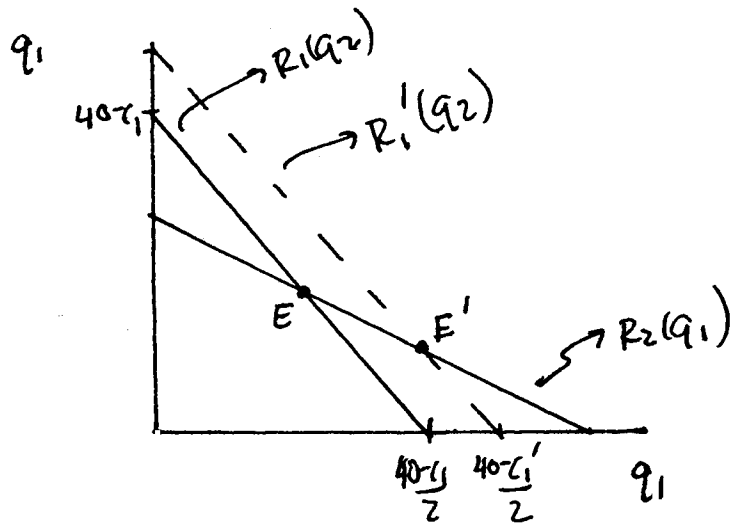
Solving:

$$\begin{aligned}
q_1 &= 20 - \frac{c_1}{2} - \frac{1}{2} \left[\frac{40 - q_1 - c_2}{2} \right] \\
q_1 &= 20 - \frac{c_1}{2} - \frac{40}{4} + \frac{q_1}{4} + \frac{c_2}{4} \\
\frac{3}{4}q_1 &= \frac{40}{4} - \frac{2c_1}{4} + \frac{c_2}{4} \\
q_1^* &= \frac{40 - 2c_1 + c_2}{3} \text{ and } q_2^* = \frac{40 - 2c_2 + c_1}{3}
\end{aligned}$$

$$\begin{aligned}
 p &= 40 - \left[\frac{40 - 2c_1 + c_2}{3} \right] - \left[\frac{40 - 2c_2 + c_1}{3} \right] \\
 &= \frac{120}{3} - \frac{80}{3} + \frac{c_1}{3} + \frac{c_2}{3} \\
 \Rightarrow p^* &= \frac{40 + c_1 + c_2}{3}
 \end{aligned}$$



(b) c_1 falls to c'_1



$q_1 \uparrow$ because $c_1 \downarrow$

$q_2 \downarrow$ according to 2's BR Function

$Q \uparrow$ since $q_1 \uparrow$ more than $q_2 \downarrow$ because slope of BR_2 is $< |1|$.

(c) With 3 symmetric Cournot Firms: (PRE-MERGER)

$$\begin{aligned}\pi_i &= \left(\frac{1-c}{N+1} \right) (p-c) \\ \Rightarrow \pi_i &= \left(\frac{1-c}{4} \right) \left(\frac{1-c}{4} \right) = \frac{(1-c)^2}{16}\end{aligned}$$

Combined, the 2 merging firms were making $\frac{(1-c)^2}{8}$ BEFORE the merger.

With 2 symmetric Cournot Firms: (POST-MERGER)

$$\pi_i = \left(\frac{1-c}{3} \right) \left(\frac{1-c}{3} \right)$$

So, the 2 merged firms make $\frac{(1-c)^2}{9}$ AFTER the merger.

$$\frac{(1-c)^2}{8} > \frac{(1-c)^2}{9} \quad \therefore \text{Profit falls.}$$

(d) When the 2 firms merge, they restrict output relative to their combined output before the merger. This would increase the price. But because Cournot best response functions are *downward sloping*, the third firm *increases* his output in response to the decrease in output by the merged firms. This increase in output reduces prices. In certain cases (i.e., the demand and cost setup here), the expansion of output by the non-merging firm can lower price enough to make the merger not profitable.

Note: Overall output falls and price increases because the \uparrow in output by the non-merging firms does not fully offset the reduction by the merging firm. But profit can still be lower because combined, the merged firms produce *less* than they did separately and if price only rises by a *little* (due to the expansion of output by the third firm) – their profits may be lower.

(e) In this case I would expect the merger to be profitable because in differentiated Bertrand, reactions functions are upward sloping. So when the merged firms raise price, the third firm will raise its price, allowing the other firms to raise their price even more until a new equilibrium is reached.