Midterm Solutions

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Question 1 a) We have only to check the expected profits net of cost for each of the two actions. Low effort produces

$$\frac{1}{3}9000 + \frac{2}{3}300 = 3200$$

while high effort produces

$$\frac{2}{3}9000 + \frac{1}{3}300 - 20 = 6080$$

The high effort level is the most profitable.

b) Now, of course, Fred must offer an incentive scheme to get the agent to work hard. The low action could be implemented at a constant wage 400, producing profits of 2800. To implement the high action we must satisfy the incentive constraint:

$$\frac{2}{3}\sqrt{m_h} + \frac{1}{3}\sqrt{m_l} - 20 \geq \frac{1}{3}\sqrt{m_h} + \frac{2}{3}\sqrt{m_l}$$

along with the participation cosntraint

$$\frac{2}{3}\sqrt{m_h} + \frac{1}{3}\sqrt{m_l} - 20 \geq 20$$

Solving these simultaneously, we obtain

$$m_l = 0 \qquad m_h = 3600$$

. Thus, from implementing the high action Fred would obtain profits

$$6100 - \frac{2}{3}3600 = 3700$$

and so implementing the high action is still optimal.

c) Now it is clear that Fred should manage the restaurant himself, earning profits $5080 > 3700$.

d) In general, we should expect risk-aversion by Fred to lead him to shift more risk to the manager, increasing the manager's incentives and getting closer to first-best. In this case, however, the IC and IR constraints are together sufficient to determine the incentive scheme, independently of Fred's utility function, so the incentive scheme does not change.

e) First, we should note that now the first-best incentives are obtainable. In particular, paying 0 for the low outcome, and 1600 for the outcome 9000, enforces e_H without imposing any risk on the agent. The payment to Fred if he hires a manger is thus $9000 - 1600 = 7400$. It is still the case that Fred should manage the restaurant himself, however, since $9000 - 1000 - 20 = 7980 > 7400$.

f) This question was really loooking for a discussion of the multi-task problem. A complete answer would say the following. If provision of quality and quantity are close substitutes in the agent's cost function, the equal compensation principle demands that quality and quantity be equally rewarded. However, since quality is hard to measure (risky), this would impose too much risk on the agent. Thus, incentives for both tasks are likely to be muted.

Question 2

Obviously, answers will vary. Almost everyone noted that increased incentives will probably increase the effort level of agents, but that this may or may not be optimal. I tended to give 5 points each for mentioning the following items:

1) The trade-off between insurance and incentives. I grouped a number of things, like risk-aversion, the cost of monitoring, uncertainty, and marginal benefit of agent effort into this category, since they all ultimately boil down to the standard moral hazard trade-off.

2) Multi-task problems like those mentioned in part f) above.

3) Issues of coordination and cooperation, like assembly-line work or inefficient competition.

4) Selection effects associated with incentive schemes such as those mentioned by Lazear.

5) Noting that agents may put in positive amounts of effort even if no incentives are given (depending on the cost of effort).

In addition, I awarded some points for particularly high-quality answers.

Question 3

a) The buyer will choose a price to maximize his utility. Note first that he will only choose among $p = 1/2$ or $p = 0$. That is, $p = 1/2$ is the minimal price that induces both types of seller's to buy, while $p = 0$ is the minimal price that gets the low-cost seller to buy. Since trade is efficient for both types of sellers, efficient trade is guranateed if the buyer chooses $p = 1/2$. He will do so if

$$(1 - 1/2) \geq q(1)$$

that is, if $q \leq 1/2$. Also, if $q = 1$, $p = 0$ is chosen but trade occurs with probability 1.

b) When the seller chooses the price, he chooses the maximum price that the buyer will accept, i.e. $p = 1$. Trade always occurs.

c) Recall that for this question we assume that the buyer's valuation is $1/2$. Thus, the buyer chooses p to maximize

$$(1/2 - p)\frac{p - x}{1/2 - x}$$

taking the derivative and setting to zero, we derive

$$p = 1/4 + x/2$$

which is less than $1/2$ if $x < 1/2$; there is a chance that efficient trade will not occur (although this probability becomes small as x gets close to $1/2$.

Again, the seller chooses $p = 1$ and there are no inefficiencies.

d) There is nothing much to say except to note that now the seller has an incentive to inefficiently exert monopoly power as well. Giving all the bargaining power to either of the parties may result in inefficiencies. However, if a third-party could enforce a trading price $p \in [1/2, 3/4]$, all types of both parties would accept, and trade would always occur.