14.22 Assignment 3 - Due Monday, March 16

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Problem 1

Consider a very simple moral hazard model. A risk-neutral principal hires an agent. The agent can take two actions, H and L. The agent's utility is given by

$$U(z, a) = z^{\frac{1}{2}} - V(a)$$

where z is how much money the agent is paid, and $V(a)$ gives the cost of action. Let $V(H) = 10$ and $V(L) = 0$. The agent's participation constraint is satisfied if $U(z, a) \geq 10$.

There are two possible cash outcomes, 1800 or 100. If the agent takes $a = H$, the outcome 1800 occurs with probability .75. If the agent takes $a = L$, the outcome 1800 occurs with probability .25.

a) Verify that in the first-best situation (with no incentive problems), the agent takes the action $a = H$ and is paid 400.

b) Verify that this contract is not incentive-compatible if the principal cannot observe the agent's action, so that the contract may depend only on the cash outcome.

c) Derive the optimal second-best contract. [Hint: What are the incentive-compatibility constraints? What is the participation constraint?]

d) Would the first-best be obtainable if the principal had access to a monitor that told him, if the outcome 100 occurred, whether the agent had taken the action L or not?

Problem 2

Recall the linear moral hazard model (i.e., exponential utility, normal disturbances, etc.). Let the agent's coefficient of risk-aversion be r, and the principal's R. The agent can exert effort e at cost $C(e) = 1/2e^2$. Output is given by

$$y = e + \epsilon$$

where ϵ is a mean zero normal random variable with variance 1. The agent must receive at least zero certain equivalent. The principal receives y dollars if y is produced.

a) Suppose the principal offers a payment scheme of the form $\alpha + \beta y$. That is, he pays the agent a fixed fee α and β dollars for every dollar of output y. Write the certain equivalents of the principal and agent for a given level of e.

b) Assume that the agent's participation constraint is binding and write an expression for the principal's certain equivalent that does not involve α. Also, find the agent's first-order condition for effort-choice.

c) Use the expressions you derived in b to derive the optimal linear contract for the principal. How does it depend on R and r?

(Hint: Parts a) and b) are easier than part c). Almost no algebra is required for those parts. The answer in c) should depend only on r and R.)