14.22 Assignment 6 - Due May 5

Jonathan Dworak

Problem 1

A buyer and a seller plan to trade with each other. The value of the good to the buyer depends on ex ante investments in quality by the seller. Given the seller's level of investment I, the value to the buyer is

$$v(I) = 3I - \frac{1}{2}I^2$$

The cost to the seller of producing the item is a fixed constant $c < 1/2$. Both parties have outside options of zero; the good is worthless to other buyers, and the buyer only values the good when produced by the seller. The investment in quality is not contractible.

a) What is the efficient level of investment?

b) What are the ex post gains from trade for a given level of I? Assume that after ex post bargaining the buyer and the seller split these gains 50-50. What level of investment will the seller choose? Compare this level to that in a); how is this effect an example of the hold-up problem?

c) What happens instead if the seller gets to choose a price at which trade takes place ex post?

Problem 2

A buyer and a seller engage in a two-period trading relationship. The buyer makes a specific investment x at cost $x^2/2$ in the first period, and receives some amount of a good from the seller in the second period. The buyer's gross benefit from the good (excluding investment costs) is $\sqrt{x}q$, where q is the amount of the good that is traded. The seller makes no investment but has ex post costs equal to $\frac{1}{2}cq^2$. The level of c is known to both parties.

a) Find the first-best level of q as a function of c, and find the first-best level of x.

b) Assume that the outside options for both parties are 0 if no trade occurs, and that parties split the ex post gains from trade 50-50. Show that inefficiencies arise.

c) Suppose instead that the buyer captures a fraction λ of the ex post gains from trade. Is there a value of λ such that the first-best can be achieved?

Problem 3

A manufacturer and worker plan to produce "wonder-widgets" with a widget-making machine. Prior to manufacturing the wonder-widget, the manufacturer must decide how much effort to expend on customizing the machine to make the widget even more wonderful. Also, the worker can engage in an investment in skill to familiarize himself with the machine. Assume that the value of the widget that is produced is given by

$$v_m m + v_w w$$

where v_m and v_w are given constants, while m and w are the investments of the manufacturer and worker, respectively. The agents' costs of investment are given by $m^2/2$, $w^2/2$.

Furthermore, whichever agent owns the machine could choose to leave the relationship and produce widgets by himself. If the manufacturer excludes the worker, the widget he produces is worth $v_m^o m$ ("v_m^o" stands for "manufacturer's value outside the rlationship"). If the worker owns the machine, he receives $v_w^o w$ outside the relationship. An agent who doesn't own the machine receives zero outside the relationship. Assume that $v_m > v_m^o$ and $v_w > v_w^o$.

a) What is the first-best level of investment for each agent?

b) Write an expression for the ex post gains from trade in the cases where the manufacturer owns the machine and where the seller owns the machine.

c) Assume that ex post bargaining results in each party being paid his outside option plus half of the gains from trade. What are the resulting investment levels in the two cases?

d) Suppose $v_m = 0$. Who should own the machine? Suppose $v_w^o = 0$. Who should own the machine?