# 14.22 Assignment 3 - Solutions

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**Problem 1**

a) In the first-best situation, we know that the agent is paid a constant wage just equal to his reservation wage. If the high action is implemented, this wage is equal to $(V(H)+10)^2 = 20^2 = 400$. If the low action is implemented similar calculations yield a wage of 100. The expected gross return for the principal if the high action is implemented is

$$0.75(1800) + 0.25(100) = 1375$$

while for the low action the payoff is

$$0.25(1800) + 0.75(100) = 525.$$

Subtracting the costs it is clear that the principal's best option is to implement the high action, paying a wage of 400 to the agent.

b) Note that the outcomes do not fully reveal which action the agent has taken. Thus the constant - wage contract will not be incentive compatible, since the low action is the cost minimizing choice for the agent. Or,

$$\sqrt{400} - 10 < \sqrt{400}.$$

c) Obviously to implement the low action it remains optimal to pay a constant wage of 100. If we wish to implement the high action, remember that both IC and IR will bind. So we must satisfy the following equations:

$$0.75\sqrt{w_{1800}} + 0.25\sqrt{w_{100}} - 10 = 0.25\sqrt{w_{1800}} + 0.75\sqrt{w_{100}}$$
$$0.75\sqrt{w_{1800}} + 0.25\sqrt{w_{100}} - 10 = 10$$

It is easy to check that $w_{1800} = 625$ and $w_{100} = 25$ solve this system, yielding an expected profit of 900 for the principal.

d) Now, the principal can make payments contigent on three possible outcomes, 1800, $\{100, H\}$ and $\{100, L\}$. For example, the principal can punish the agent severely if $\{100, L\}$ occurs, and pay 400 in the other two outcomes. Since when the agent takes $H$ the outcome $\{100, L\}$ is avoided, this imposes no risk on the agent, but still gives him incentives to take the high action. For concreteness note that paying 0 in the state $\{100, L\}$ and 400 otherwise satisfies all constraints.

**Problem 2**

All the mumbo-jumbo about "exponential utilities" and "normal random variables" just means that you can write the certain equivalents in the simple mean-variance form that was used in class.

a)The mean of the principal's payoff is $e - \alpha - \beta e = -\alpha + (1-\beta)e$. Since $\alpha$ is a constant, the variance of this payoff is $(1-\beta)^2\sigma^2$. Thus, the principal's CE is

$$-\alpha + (1-\beta)e - \frac{1}{2}R(1-\beta)^2\sigma^2$$

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The agent's CE is given by the standard expression:

$$\alpha + \beta e - \frac{1}{2}e^2 - \frac{1}{2}r\beta^2\sigma^2$$

b) If the agent's CE $= 0$, we can solve for $\alpha$ and substitute into the principal's CE:

$$e - \frac{1}{2}e^2 - \frac{1}{2}r\beta^2\sigma^2 - \frac{1}{2}R(1-\beta)^2\sigma^2$$

The agent's FOC is the $\beta = e$

c) Plug in $\beta = e$ to the principal's objective function:

$$\beta - \frac{1}{2}\beta^2 - \frac{1}{2}r\beta^2\sigma^2 - \frac{1}{2}R(1-\beta)^2\sigma^2$$

and we can just maximize over $\beta$. Taking the derivative and setting it equal to zero, we can derive:

$$\beta = \frac{1 + R\sigma^2}{1 + (r + R)\sigma^2}$$

This also gives us $e$ and we can solve to get an expression for $\alpha$.

What should we note about this contract? Again, if $r = 0$, we get first-best incentives. More interestingly, if $R$ is very large, first-best incentives are optimal. That is, if the principal is very risk-averse relative to the agent, it is optimal for the agent to bear all of the risk. Since this is also consistent with giving the agent high-powered incentives, things work out very nicely.