How to Solve Hold-up Problems

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Hold-up - General Case

As we saw in class, there are two components to any hold-up problem. The first is that the parties involved are working with *specific* assets that are more valuable in the relationship than outside. This creates the potential for one party to hold-up another by threatening to withhold access to the asset, or payment for the use of the asset, or whatever. The upshot is that the parties must bargain ex post over how the returns from the use of the specified asset are to be divided.

In general, however, it is assumed that this bargaining implements the efficient use of the asset; since the bargaining is just a division of the rents, there is no problem ex post. The inefficiency comes about when there are *ex ante* specific investments that the parties must undertake, which investments themselves are not contractible. Because of the ex post hold-up, the parties know that they will not be able to capture the full returns from their investments. This leads to under-investment, and thus ex ante inefficiency.

Thus solving the problems involves three stages:

1) Determine the ex post optimal decision, given the level of investments.

2) Determine the ex post division of the surplus; this will depend on the parties' outside options, but not on the cost they incurred in investing (this cost is sunk).

3) Given each agent's final payoff, determine the level of investment he will undertake.

Let's consider a simple 2-agent model, and assume that part 1) has been solved, leading to a function $V(I_1, I_2)$ which represents the total surplus as a function of the agents' investment levels. Suppose also that the agents have outside options given by $V_1(I_1, I_2)$ and $V_2(I_1, I_2)$. That is, these are the payoffs that the agents would receive if they left the relationship. (I have written these outside options as if they depend on investments; of course it's possible that they don't). Since the agents are working with specific assets, we assume $V \geq V_1 + V_2$; it is efficient for the agents to work together. The gains from trade are therefore $V - V_1 - V_2$.

There is no general theory about how the agents will divide this surplus. A simple solution is to assume that each agent gets his outside option plus some fraction (say 1/2) of the gains from trade. Thus, in this example, agent 1 receives: $\frac{1}{2}[V - V_1 - V_2] + V_1$, similarly for agent 2. With this solution, at the investment stage agent 1 chooses I_1 to maximize $\frac{1}{2}[V - V_1 - V_2] + V_1 - C(I_1)$, where $C(I_1)$ represents the cost of choosing investment level I_1.

A Specific Example

A buyer and a seller face each other in a relationships. If the two parties do not trade, they receive 0. The value of the good to the buyer is given by xv, where v is a constant and x is the buyer's ex ante investment in the relationship. The seller's cost is xc. The cost of investment for the buyer is $x^2/2$. Assume that $v - c > 0$. The buyer and seller split the gains from trade 50-50. I'll go through the steps in order

1) Given x, the only decision to be made is whether to trade or not. Trade is efficient if $xv > xc$, which is always satisfied given $v - c > 0$. Thus, the ex post total surplus is $x(v - c)$.

2) The outside options of each party are 0, so final payoffs to each party are given by $\frac{1}{2}x(v - c)$.

3) The seller has no investment to make. The buyer maximizes

$$\frac{1}{2}x(v-c)-\frac{x^2}{2}$$

which gives $x^* = \frac{1}{2}(v-c)$.

What is the first-best level of investment? This is the x that maximizes total surplus: $x(v-c)-\frac{x^2}{2}$, i.e. $x^* = v-c$. We get the familiar underinvestment effect because the seller expropriates some of the ex post rents.

Ownership

Ownership enters these problems insofar as it affects outside options. For example, the owner of a physical asset may be able to sell it if the relationship breaks down. Since outside options affect final payoff, which in turn affects the incentive to invest, we see that ownership can affect the ex ante efficiency of investments. In these cases we have only to fix the various ownership structures, and solve for the resulting investment levels. Then we can compare investment levels accross ownership patterns to determine the efficient ownership structure.