

Problem Set 3 Answers

Question A1

(a) The payoff matrix is

| | | | |
|----------|-------|----------|-------|
| | | Player B | |
| | | Heads | Tails |
| Player A | Heads | 1,-1 | -1,1 |
| | Tails | -1,1 | 1,-1 |

Clearly there is no pure-strategy Nash equilibrium. In any candidate equilibrium, either the coins match or they don't. If the coins match, B would want to flip sides. If they don't, A would flip sides.

(b) Let α be the probability that A plays Heads (thus $1 - \alpha$ be the probability that A plays Tails) and β be the probability that B plays Heads (thus $1 - \beta$ is the probability B plays Tails).

Given α , B must be indifferent between Heads and Tails.

- If B plays Heads, obtains $(-1)(\alpha) + (1)(1 - \alpha) = 1 - 2\alpha$. (I)
- If B plays Tails, obtains $(1)(\alpha) + (-1)(1 - \alpha) = 2\alpha - 1$. (II)

For B to be indifferent, (I)=(II), implying $1 - 2\alpha = 2\alpha - 1$, implying $\alpha^* = 1/2$.

Given β , A must be indifferent between Heads and Tails.

- If A plays Heads, obtains $(1)(\beta) + (-1)(1 - \beta) = 2\beta - 1$. (III)
- If A plays Tails, obtains $(-1)(\beta) + (1)(1 - \beta) = 1 - 2\beta$. (IV)

For A to be indifferent, (III)=(IV), implying $2\beta - 1 = 1 - 2\beta$, implying $\beta^* = 1/2$.

The Nash equilibrium is for both players to randomize over Heads and Tails with probability 1/2. As an exercise, compute what the players' equilibrium expected payoffs are.

Question A2

(a) We can rule out two possibilities: both choosing Chicken (C) and both choosing Not Chicken (NC) cannot be Nash equilibria. Fonzi, for instance, would have an incentive to deviate from either proposed equilibrium. The remaining two possibilities are Nash equilibria.

(b) Let f be the probability Fonzi plays C and p be the probability Pinkie plays C. Given f , Pinkie must be indifferent between C and NC.

- If Pinkie plays C, obtains $(2)(f) + (1)(1 - f) = 1 + f$. (I)
- If Pinkie plays NC, obtains $(3)(f) + (0)(1 - f) = 3f$. (II)

For Pinkie to be indifferent, (I)=(II), implying $1 + f = 3f$, implying $f^* = 1/2$. The game is symmetric, so Pinkie choice $p^* = 1/2$ makes Fonzi indifferent.

(c.i) Fonzi’s strategy is C or NC as before. Pinkie’s strategy is contingent on Fonzi’s action. It is an ordered pair (a, b) where Pinkie moves a if Fonzi chooses C and b if Fonzi chooses NC. The possible strategies are thus

- (C,C): always chicken out (wimpy)
- (C,NC): follow the leader (dumb)
- (NC,C): do the opposite (pragmatic)
- (NC,NC): always drive straight (tough).

(c.ii) The payoff matrix is

| | | Pinkie | | | |
|-------|----|--------|--------|--------|---------|
| | | (C,C) | (C,NC) | (NC,C) | (NC,NC) |
| Fonzi | C | 2,2 | 2,2 | 1,3 | 1,3 |
| | NC | 3,1 | 0,0 | 3,1 | 0,0 |

(c.iii) There are three Nash equilibria:

- Fonzi chooses NC, Pinkie chooses (C,C) (wimpy)
- Fonzi chooses NC, Pinkie chooses (NC,C) (pragmatic)
- Fonzi chooses C, Pinkie chooses (NC,NC) (tough)

(c.iv) The “wimpy” equilibrium isn’t subgame perfect. In the subgame in which Fonzi chooses C, it isn’t rational for Pinkie to choose C, too. The “tough” equilibrium isn’t subgame perfect either. In the subgame in which Fonzi chooses NC, it isn’t rational for Pinkie to choose NC, too.

Question B1

(a) There is only one Nash equilibrium: McGee doesn't pay, and the contractor doesn't build. This equilibrium is in dominant strategies.

(b) The first step is to write down the normal form for this sequential game (note the contractor is player 1 here):

| | | | | | |
|------------|-------|----------------|----------------|-------------|---------------|
| | | McGee | | | |
| | | (Pay,Pay) | (Pay,Don't) | (Don't,Pay) | (Don't,Don't) |
| Contractor | Build | $p - c, v - p$ | $p - c, v - p$ | $-c, v$ | $-c, v$ |
| | Don't | $p, -p$ | $0, 0$ | $p, -p$ | $0, 0$ |

In the sole Nash equilibrium, the contractor does not build and McGee never pays (Don't,Don't). This equilibrium is also subgame perfect.

(c)

| | | | | | |
|-------|-----------|----------------|----------------|---------------|---------------|
| | | Contractor | | | |
| | | (Build,Build) | (Build,Don't) | (Don't,Build) | (Don't,Don't) |
| McGee | Pay | $v - p, p - c$ | $v - p, p - c$ | $-p, p$ | $-p, p$ |
| | Don't Pay | $v, -c$ | $0, 0$ | $v, -c$ | $0, 0$ |

In the sole Nash equilibrium, McGee doesn't pay and the contractor never builds (Don't,Don't). This is also the subgame perfect Nash equilibrium.

(d) For McGee not to deviate from the proposed equilibrium, (Pay,Build), $v - p \geq v - B_m$, implying $B_m \geq p$. For the contractor not to deviate from the proposed equilibrium, $p - c \geq p - B_c$, implying $B_c \geq c$. If these inequalities are strict, then (Pay,Build) is the unique Nash equilibrium.

Question B2

(a) In general, A's objective function can be written $\alpha(4r_a^{2/4}r_b^{1/4}) - r_a$. If $\alpha = 0$, there is no benefit from reliance, only an expense, so $r_a^* = 0$. Similarly, $r_b^* = 0$ if $\alpha = 0$.

(b) A chooses r_a to maximize $4r_a^{2/4}r_b^{1/4} - r_a$. The first-order condition is

$$(4) \left(\frac{2}{4}\right) r_a^{-2/4} r_b^{1/4} - 1 \\ \implies 16r_b = r_a^2. \quad (1)$$

B chooses r_b to maximize $4r_a^{1/4}r_b^{2/4} - r_b$. The first-order condition is

$$(4) \left(\frac{2}{4}\right) r_a^{1/4} r_b^{-2/4} - 1 \\ \implies r_a = \frac{r_b^2}{16} \\ \implies r_a^2 = \frac{r_b^4}{16^2}. \quad (2)$$

Substituting (2) into (1) yields $16r_b = r_b^4/16^2$, implying $16^3 = r_b^3$, in turn implying $r_b^* = 16$. Substituting, $r_a^* = 16$ as well.