14.24
Prof. Snyder
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Problem Set 4 Answers

Question 1

Choose x to maximize

$$wx + p(x)A = 1,000x + 100,000e^{-x}.$$

The first-order condition is

$$\begin{aligned} 1,000 - 100,000e^{-x} &= 0 \\ \Longrightarrow \quad e^{-x} &= \frac{1}{100} \\ \Longrightarrow \quad e^{x} &= 100 \\ \Longrightarrow \quad x &= \ln 100. \end{aligned}$$

Hence, $x^* \approx 4.6$.

Question 2

(a) Total social cost equals

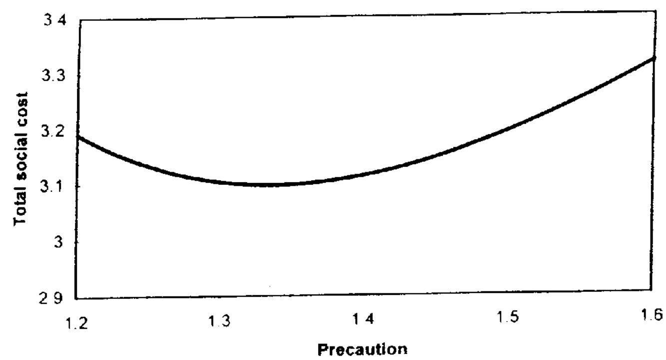
$$\begin{aligned} SC(x_v, x_i) &= x_v x_v + w_i x_i + p(x_v, x_i)A \\ &= x_v + x_i + 100\exp(-(x_v+1)(x_i+1)). \end{aligned}$$

Noting that $x_v = x_i$ at an optimum,

$$SC(x_v, x_i = x_v) = 2x_v + 100\exp(-(x_v+1)^2).$$

Figure 1 presents a graph of this function. Total social cost is minimized for $x_v^{s.o.} = x_i^{s.o.} \approx 1.35$. At these values, $SC^{s.o.} \approx 3.1$.

Figure 1

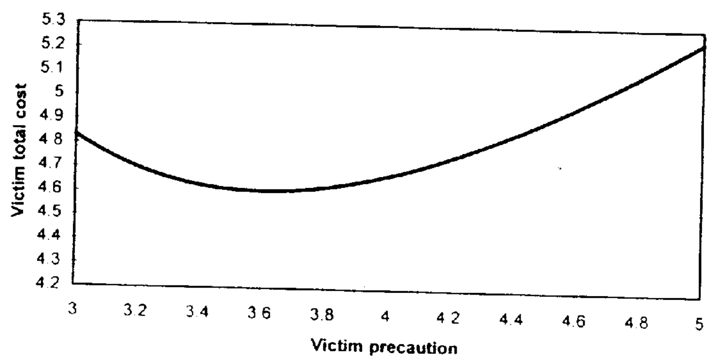


(b) Precaution provides no benefit to the injurer, so $x_i^* = 0$. Given this, the victim solves

$$\min_{x_v}\{w_v x_v + 100\exp(-x_v + 1))\},$$

which is its personal-cost function with $x_i = 1$ substituted. Figure 2 presents a graph of this objective function: it is minimized at $x_v^* \approx 3.6$. Substituting x_v^* and x_i^* into the total social cost function, $SC^* \approx 4.6$.

Figure 2



Social cost is higher in this equilibrium than in the social optimum. The injurer's precaution is lower and the victim's, to compensate, is higher than in the social optimum.

(c) This problem is the mirror image of (b) above with the labels "injurer" and "victim" switched. Hence, $x_v^* = 0$, $x_i^* \approx 3.6$, and $SC^* \approx 4.6$.

(d) Here we have

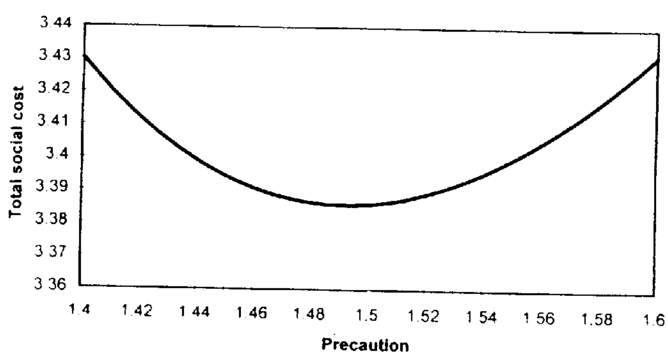
$$SC(x_v, x_i = x_v) = 2x_v + 200\exp(-(x_v + 1)^2)$$

rather than

$$SC(x_v, x_i = x_v) = 2x_v + 100\exp(-(x_v + 1)^2)$$

as in (a) above. Figure 3 presents a graph of the function. It is minimized at $x_v^{s.o.} = x_i^{s.o.} \approx 1.5$, at which values $SC^{s.o.} \approx 3.4$. An increase in A is thus shown to increase the socially-optimal levels of precaution.

Figure 3



Question 3

(a) The victim chooses a_v to solve

$$\max_{a_v}\{a_v - a_v^2\}.$$

The first-order condition is $1 - 2a_v = 0$, implying $a_v^* = 1/2$. Similarly, $a_i^* = 1/2$.

(b) The social objective function is

$$\max_{a_v,a_i}\{u_v(a_v)+u_i(a_i)-p(a_v,a_i)A\}$$

or, upon substituting,

$$\max_{a_v,a_i}\{a_v-a_v^2+a_i-a_i^2-a_va_i\}.$$

The first-order condition for the choice of a_v is

$$1-2a_v-a_i=0 \tag{1}$$

and for the choice of a_i is

$$1-2a_i-a_v=0. \tag{2}$$

Equations (1) and (2) are two linear equations in two unknowns. Solving using standard techniques (for example, solve (1) for a_i, substitute into (2), and solving for a_v) yields $a_v^{s.o.}=a_i^{s.o.}=1/3$. Substituting these values into the welfare function yields $W^{s.o.}=1/3$.

(c) *No Liability.* The injurer behaves as if there were no accidents and so chooses the same level of activity as in (a) above: $a_i^*=1/2$. The victim solves

$$\begin{aligned}&\max_{a_v}\{u_v(a_v)-p(a_v,a_i=1/2)A\}\\ =\;&\max_{a_v}\{a_v-a_v^2-\frac{a_v}{2}\}.\end{aligned}$$

The first-order condition is $1-2a_v-1/2=0$, implying $a_v^*=1/4$. Welfare is $W^*=5/16$. In sum, the injurer over-indulges in the activity relative to the social optimum, and this reduces social welfare; the victim chooses a lower activity level than in the social optimum to compensate.

Strict Liability. This is the mirror image of the no liability case with the labels of "victim" and "injurer" switched. Here, $a_v^*=1/2$, $a_i^*=1/4$, and $W^*=5/16$.

Negligence. Since there is no precaution involved in the model, the injurer can be assumed to satisfy the standard trivially. Thus, the negligence regime is identical to the no-liability regime. (If you assume, alternatively, that the absence of precaution means that the injurer cannot satisfy the standard, then the negligence regime is equivalent to the strict liability regime.)

Question 4

With the modification, Table 8.4 is given below. First, suppose consumers are perfectly informed.

Table 8.4
COST OF SODA

Behavior of firm	*Firm's cost of production per unit*	*Probability of accident to consumer*	*Loss if accident*	*Expected accident loss*	*Full cost per unit*
	(1)	(2)	(3)	(4)	(5)
Use bottle	40 cents	1/100,000	$4,000	4 cents	44 cents
Use can	43 cents	1/200,000	$4,000	2 cents	45 cents

Under a no-liability rule, consumers will pay 40 cents for bottles and 43 cents for cans. Consumers also pay expected accident costs, but these are not enough to make cans cheaper than bottles in terms of full cost (44 cents for bottles compared to 45 for cans). Only bottles will be produced in a perfectly competitive equilibrium. Under a strict-liability rule, the full cost of the containers will be reflected in the price paid by consumers; firms will pass expected liability costs through to consumers. In equilibrium, only bottles will be produced, and they will be sold for 44 cents.

Second, suppose consumers are imperfectly informed. Under a no-liability rule, consumers may purchase cans if they overestimate the probability of accident with bottles relative to that for cans. This may lead to inefficiencies. A strict-liability rule causes the price to reflect the actual probabilities of accident, so that the equilibrium price for bottles is 44 cents. The equilibrium price for cans if they were sold on the market would be 45 cents, but in equilibrium they wouldn't even be sold. Thus a strict-liability rule is efficient whether or not consumers are informed; a no-liability rule may not be.