Elasticities in Two-Stage Demand Systems

This handout shows how to calculate demand elasticities in the two-stage demand system used by Hausman and Leonard (1997).

Brand-level demand:
The revenue share of brand \(i\), \(s_i\), is given by the following equation, where city and time subscripts have been suppressed:

\[
s_i = \alpha_i + \beta_i \log \frac{Y}{P} + \gamma_{ij} \log p_j + \epsilon_i
\]

\(Y\) is total industry expenditure, and \(P\) is a price index given by \(\log P = \sum s_k \log p_k\).

Industry-level demand:
Real industry expenditure \((u = Y/P)\) is given by:

\[
\log u = \mu + \delta_1 \log y + \delta_2 \log P + Z\delta_3 + \epsilon
\]

There are two types of elasticities: conditional elasticities, which hold industry expenditure \((Y)\) constant, and unconditional elasticities, which take into account the fact that industry expenditure changes when brand prices change.

Calculating conditional elasticities:

\[
q_i = s_i Y / p_i
\]

\[
\log q_i = \log s_i + \log Y - \log p_i
\]

\[
\frac{\partial \log q_i}{\partial \log p_j} = \frac{\partial \log s_i}{\partial \log p_j} - 1(i = j)
\]

where \(1(.)\) is the indicator function and since \(Y\) is held constant.

\[
\frac{\partial \log s_i}{\partial \log p_j} = \frac{1}{s_i} \frac{\partial s_i}{\partial \log p_j} = \frac{1}{s_i} \left( -\beta_i s_j + \gamma_{ij} \right), \text{ since } \frac{\partial \log P}{\partial \log p_j} = s_j
\]

Hence the conditional elasticity of demand for brand \(i\) w.r.t. the price of brand \(j\) is:

\[
\frac{\partial \log q_i}{\partial \log p_j} \bigg|_{y=y_j} = \frac{1}{s_i} \left( -\beta_i s_j + \gamma_{ij} \right) - 1(i = j)
\]
To derive the unconditional elasticities, we need to recalculate \( \frac{\partial \log s_i}{\partial \log p_j} \) and \( \frac{\partial \log Y}{\partial \log p_j} \): 

\[
\frac{\partial \log s_i}{\partial \log p_j} = 1 \frac{\partial (\alpha_i + \beta_i (\mu + \delta_i \log y + \delta_2 \log P + Z\delta_3 + \epsilon_i) + \sum_j \gamma_{ij} \log p_j + \epsilon_i)}{\partial \log p_j} = \frac{1}{s_i} (\beta_i \delta_2 s_j + \gamma_{ij})
\]

\[
\frac{\partial \log Y}{\partial \log p_j} = \frac{\partial \log u}{\partial \log p_j} + \frac{\partial \log P}{\partial \log p_j} = (\delta_2 + 1)s_j
\]

So the unconditional elasticity of demand for brand \( i \) w.r.t. the price of brand \( j \) is:

\[
\frac{\partial \log q_i}{\partial \log p_j} = \frac{1}{s_i} (\beta_i \delta_2 s_j + \gamma_{ij}) + (\delta_2 + 1)s_j - 1(i = j)
\]