PROBLEM SET 2: RELATIONAL CONTRACTS
"DUE" FRIDAY SEPTEMBER 24

Problem 1: Efficiency wages vs. subjective bonuses

Consider the following stage game of an efficiency-wage model. The agent can choose either high effort, a_H, or low effort, a_L. High effort yields high output, y, with probability one, whereas low effort yields high output with probability p but zero output with probability 1-p. The agent is risk-neutral, with payoff w - c(a), where w is the wage earned and $c(a_H) = c > c(a_L) = 0$. The principal is risk-neutral, with payoff equal to profit—namely, output minus wages.

The timing of this stage game is: (1) the principal offers the wage w; (2) the agent accepts or rejects (in favor of alternative employment with payoff U_0); if the agent accepts then (3) the principal pays w; (4) the agent chooses a_H or a_L (but the principal does not observe this choice); (5) output is observed by the principal and the agent (but not by a court); and (6) if output is low (0) then the agent is fired, earning U_0 every period thereafter. Assume that $y - c > U_0 > py$, so that high effort is efficient. Finally, let the interest rate be r.

State trigger strategies that would achieve high effort and high output in every period, and that are a subgame-perfect Nash equilibrium of the repeated game if the following condition holds:

(*) $$y - c \geq U_0 + \frac{r}{1-p} c.$$

Now consider the following timing of a new stage game, to be played in the same economic environment as above (*i.e.*, under the same assumptions about feasible actions, the relationship between actions and outputs, and so on): (1) the principal offers the contract (s, b); (2) the agent accepts or rejects (in favor of alternative employment with payoff U_0); if the agent accepts then (3) the principal pays the salary s; (4) the agent chooses a_H or a_L (but the principal does not observe this choice); (5) output is observed by the principal and the agent (but not by a court); and (6) if output is high then the principal chooses whether or not to pay the bonus b.

State trigger strategies that would achieve high effort, high output, and a bonus paid in every period. Show that these trigger strategies are a subgame-perfect Nash equilibrium of the repeated game if (*) holds.

Problem 2: Stationary relational contracts

Prove Theorem 2 from Levin (2003), for the moral-hazard version of Levin's model that we discussed in class: If there is an optimal relational contract then a stationary relational contract is optimal.

Problem 3: Relational Contract Meets Multitask

This problem (eventually) concerns objective and subjective performance measurements in a multi-task relational incentive problem.

In each period, the environment is as follows (where time subscripts are omitted for simplicity). The value to the Principal from the Agent's actions (a_1, a_2) is $y = y_H$ or y_L $(< y_H)$, where y is observable but *not* contractible. The probability that $y = y_H$ is $f_1a_1 + f_2a_2$, where f_1 and f_2 are non-negative and small enough that $f_1a_1 + f_2a_2 < 1$. The Agent's cost function is $c(a_1, a_2) = [a_1^2 + a_2^2] / 2$. If w is the total compensation paid to the Agent in a given period then the Principal's payoff in that period is y – w and the Agent's is w - $c(a_1, a_2)$. Both parties are risk-neutral, have deep pockets, and share the discount rate r. The Principal's reservation payoff is π_0 in each period and the Agent's is U_0, where $\pi_0 + U_0 > y_L$.

(a) What is the first-best action vector?

(b) Consider the infinitely repeated game in which the stage game has the following timing: (i) Principal and Agent can exchange money; (ii) Agent chooses actions (but Principal cannot observe them); (iii) y is publicly observed; (iv) Principal and Agent can exchange money. Specify trigger strategies that, if played, will yield the first-best. (For notational consistency in what follows, use a subjective bonus scheme that pays w = s if $y = y_L$ but w = s + B if $y = y_H$.) For what values of r (given the other parameters) are your strategies a subgame-perfect Nash equilibrium of the repeated game?

Now enrich the stage game to include the performance measure $p = p_H$ or p_L $(< p_H)$, where p *is* contractible. The probability that $p = p_H$ is $g_1a_1 + g_2a_2$, where g_1 and g_2 are non-negative and small enough that $g_1a_1 + g_2a_2 < 1$.

(c) Consider the one-shot agency problem (*i.e.*, not yet a repeated game) in which the Principal's payoff is y – w and the Agent's is w - $c(a_1, a_2)$, where y is not contractible but p is. Consider the incentive contract w = s if $p = p_L$ but w = s + b if $p = p_H$. What is the efficient value of b? Let b* denote the efficient value of b and let $E\pi(s, b^*)$ denote the resulting expected payoff to the Principal when the salary is s. Suppose that the parties determine s via Nash bargaining, where the Principal's bargaining power is $\alpha \in (0, 1)$.

Denote the resulting salary by s_α, the Principal's expected payoff by $E\pi(s_\alpha, b^*)$, and the Agent's by $EU(s_\alpha, b^*)$.

(d) Now consider the infinitely repeated game in which the stage game has the following timing: (i) Principal and Agent can contract on p; (ii) Principal and Agent can exchange money; (iii) Agent chooses actions (but Principal cannot observe them); (iv) y and p are publicly observed; (v) contracts based on p are enforced; (vi) Principal and Agent can exchange money. Assume that if reneging occurs in the repeated game then the parties will play the efficient one-shot contract from (c) forever after, where the parties' expected payoffs are $E\pi(s_\alpha, b^*)$ and $EU(s_\alpha, b^*)$. Specify trigger strategies that, if played, will yield the first-best. For what values of r (given the other parameters) are your strategies a subgame-perfect Nash equilibrium of the repeated game? Are these values of r higher or lower than in (b), and why?

(e) Continue to consider the repeated game from (d), including the assumption of efficient one-shot contracting after reneging. If r is sufficiently high that the first-best cannot be achieved, it is natural to consider other relational contracts that attempt to outperform the efficient agency contract in (c). Consider the following incentive scheme: $w = s$ if $p = p_L$ and $y = y_L$, $w = s + b$ if $p = p_H$ and $y = y_L$, $w = s + B$ if $p = p_L$ and $y = y_H$, and $w = s + \beta$ if $p = p_H$ and $y = y_H$. Suppose that $f_1 = g_1 > 0$, $f_2 = 0$, and $g_2 > 0$. Are there finite values of r such that the only relational-contract outcome is the trivial one, which replicates the efficient agency contract from (c)? Why or why not?