PROBLEM SET 3: CAREER CONCERNS
"DUE" FRIDAY OCTOBER 1.

Problem 1: Normal Learning Model

Suppose that z_t is a noisy signal about η, $z_t = \eta + \varepsilon_t$, where η is Normally distributed with mean m_0 and variance $1/h_0$ (*i.e.*, precision h_0) and the ε_t are i.i.d. with mean zero and precision h_ε and are independent of η. Show that

$$E(\eta \mid z_1) = (h_0 m_0 + h_\varepsilon z_1) / (h_0 + h_\varepsilon)$$

and then that

$$E(\eta \mid z_1, \ldots, z_t) = [h_0 m_0 + h_\varepsilon(z_1 + \ldots + z_t)] / (h_0 + t h_\varepsilon) .$$

Problem 2: Multi-task Meets Career Concerns

Consider the following two-period multi-task incentive problem:

1) the Agent's (observable but non-contractible) output in period t is $y_t = \eta + f_1 a_{1t} + f_2 a_{2t} + \varepsilon_t$;
2) the Agent's measured (and contractible) performance in period t is $p_t = \eta + g_1 a_{1t} + g_2 a_{2t} + \phi_t$;
3) the prior on the Agent's ability, η, is Normally distributed with mean m and precision h;
4) the error terms ε_t and ϕ_t are Normally distributed with mean zero and precisions h_ε and h_ϕ, respectively, and are independent of each other and of η;
5) the Agent's action cost in period t is $c(a_{1t}, a_{2t}) = [a_{1t}^2 + a_{2t}^2] / 2$;
6) the Agent's payoff is $U = [w_1 - c(a_{11}, a_{21})] + \delta[w_2 - c(a_{12}, a_{22})]$, and the Principal's payoff is $\pi = [y_1 - w_1] + \delta[y_2 - w_2]$;
7) the parties are risk-neutral;
8) the wage contract in period t is $w_t = s_t + b_t p_t$;
9) there is perfect competition among identical Principals in each period; and
10) the parties cannot commit in period 1 to any actions in period 2.

Solve for the optimal contracts in periods 1 and 2. Identify and describe the terms that make b_1^* different from b_2^*.

Problem 3: Influence Activities

Milgrom and Roberts (1988) argue that an important cost of integration is influence (*i.e.*, when you give someone authority, she gets lobbied). This problem investigates that idea using a career-concerns model.

The timing is: (i) two parties negotiate over control of the decision right; (ii) the parties simultaneously choose influence actions, with party i choosing action $a_i \in A$ at cost $c(a_i)$; (iii) the parties publicly observe the signal σ; (iv) the party with control chooses a decision, $d \in D$; (v) the parties receive their payoffs, $U_i(s, d)$ for i = 1, 2 (where U_i is gross of any monetary transfers and action costs).

Let $U_i(s, d) = - k_i (d - s - b_i)^2$, where $k_i > 0$ and $b_i \in \Re$. The parameter k measures the party's sensitivity to the difference between the decision taken and that party's ideal decision; the parameter b measures how the party's ideal decision differs from the state.

Let $\sigma = s + a_i + a_j + \varepsilon$, and let there be symmetric uncertainty: both parties share the prior belief that s is Normal with mean m and variance 1/h (*i.e.*, precision h) and ε is Normal with mean zero and precision h_ε (and independent of s). Let $D = \Re$ and $A = \Re$, and let the cost function be symmetric around zero, with $c'(0) = 0$, $c'(-\infty) = -\infty$, $c'(\infty) = \infty$, and $c'' > 0$.

Suppose party i is given control in stage (i).

(a) What problem does party i solve in stage (iv)?
(b) What is that problem's solution?
(c) What problem does party j solve in stage (ii)?
(d) What is that problem's solution?
(e) How does party j's equilibrium influence activity vary with k_j?
(f) What is the expected total payoff from allocating control to party i?

Problem 4

A tenant works with a machine owned by an entrepreneur to produce one widget per period at a periodic cost of

$$c_t(e_t,\beta) = \beta - e_t - \varepsilon_t\,, \quad t = 1,2.$$

Here e_t is the amount of unobserved effort that the tenant puts in to lower the unit cost in period t; ε_t is a transitory disturbance term, which is distributed Normally with mean zero and precision (inverse of variance) h_ε; and β is a cost characteristic of the machine that neither the tenant nor the entrepreneur knows with certainty.

Both hold symmetric beliefs about β, viewing it at the start as Normally distributed with mean b_0 and precision h_0 and updating these beliefs based on the mutually observed realization of c_1. The disturbance terms ε_t are independent of each other and the cost parameter β.

The tenant earns p in each period from selling the widget in a competitive market. The tenant bears all costs of producing the widget (the observed, but unverifiable cost c_t as well as a private, unobserved cost of effort $g(e_t)$; $g' > 0$, $g'' < 0$).

Each period the tenant pays the entrepreneur in advance a rental fee r_t set so that the tenant earns a constant net *expected* income in that period (which you can take to be zero). As in the model of career concerns, the periodic rental is based on a forecast of the tenant's effort decision and the beliefs currently held about the uncertain cost parameter of the machine, β.

a. Write down the conditions characterizing a Nash equilibrium of this model. (For this part, don't make use of specific distributional assumptions.)

b. Show that the tenant's effort level will be inefficiently low in the first period and optimal in the second period.

c. Explain why the *increasing* effort pattern in part (b) is so different from the *decreasing* effort pattern in the standard career-concerns model.