PROBLEM SET 4: DECISION RIGHTS AND EX ANTE INCENTIVES
"DUE" FRIDAY OCTOBER 8.

Problem 1: Incentive-System Theory of the Firm

Consider the following multi-task incentive problem: the Agent's (non-contractible) output is $y = f_1a_1 + f_2a_2 + \varepsilon$; the Agent's measured performance is $p = g_1a_1 + g_2a_2 + \phi$; wage contracts are $w = s + bp$; and the Agent's action costs are $c(a_1, a_2) = [a_1^2 + a_2^2] / 2$.

As in Holmstrom-Milgrom (JLEO 91), there is a machine that the Agent uses in producing y. The resale value of the machine (after it is used in production) is $v = h_1a_1 + h_2a_2 + \xi$. If the Principal owns the machine (so the Agent is an "employee"), then the Principal's payoff is $y + v - w$ and the Agent's is $w - c$. If the Agent owns the machine (so the Agent is an "independent contractor"), then the Principal's payoff is $y - w$ and the Agent's is $w + v - c$. The parties are risk-neutral.

a) Solve for the efficient contract slope, b_E^*, when A is an employee. What is the expected total surplus, S_E, from this efficient contract?

b) Solve for the efficient contract slope, b_I^*, when A is an independent contractor. What is the expected total surplus, S_I, from this efficient contract?

c) Provide sufficient conditions on f, g, and h such that $S_E > S_I$. Explain. Provide sufficient conditions on f, g, and h such that $S_I > S_E$. Explain.

Problem 2: Multi-task Property-Rights Theory of the Firm

Consider the following Principal-Agent model. The timing is as in Grossman and Hart's (1986) model of integration: (1) the parties allocate control of a single decision right, $d \in D$; (2) the Agent chooses an action $a \in A$; (3) the action and the state of the world ($s \in S$) are publicly observed; (4) the decision is chosen; and (5) the payoffs $U_P(a, s, d)$ and $U_A(a, s, d) - c(a)$ are received by the Principal and the Agent, respectively, where c(a) is the Agent's cost function.

The allocation of the decision right is contractible in stage (1). The (multi-task) action $a = (a_1 , a_2)$ is not contractible at any point, and the Agent's cost function is $c(a_1, a_2) = [a_1^2 + a_2^2] / 2$, where $a_i \geq 0$. The decision $d \in D$ is contractible in stage (4) but not before that. In stage (4), the parties choose the decision via Nash bargaining, with equal bargaining powers. The parties' net payoffs are their gross payoffs received in stage (5), plus or minus any transfers made in stages (1) or (4), minus c(a) for the Agent. The parties are risk-neutral and have deep pockets.

The states are $s \in \{1, \dots, S\}$. The decisions are $d \in \{d_{P1}, d_{A1}, \dots, d_{Ps}, d_{As}, \dots, d_{PS}, d_{AS}\}$. For a given (a, s), the relevant decisions are d_{Ps} and d_{As}, as follows:

$$U_P(a, s, d_{Ps}) = f_1 a_1 + f_2 a_2 + K_P;$$

$$U_A(a, s, d_{Ps}) = 0;$$

$$U_P(a, s, d_{As}) = g_1 a_1 + g_2 a_2;$$

$$U_A(a, s, d_{As}) = h_1 a_1 + h_2 a_2 + K_A; \text{ and}$$

$$U_P(a, s, d) = U_A(a, s, d) = 0 \text{ for all } d \notin \{d_{Ps}, d_{As}\}.$$

Assume that f_i, g_i, and $h_i \geq 0$. Assume also that $K_P > 0$ is sufficiently large that $U_P(a, s, d_{Ps}) > U_P(a, s, d_{As})$ for all relevant (a, s), but that $K_A > 0$ is sufficiently large that $U_A(a, s, d_{As}) + U_P(a, s, d_{As}) > U_P(a, s, d_{Ps})$ for all relevant (a, s).

(a) Given (a, s), what is the efficient decision $d \in \{d_{P1}, d_{A1}, \ldots, d_{Ps}, d_{As}, \ldots, d_{PS}, d_{AS}\}$?

(b) If the Agent is allocated the decision right in stage (1), what actions will she choose in stage (2), and what will be the parties' resulting total surplus?

(c) If the Principal is allocated the decision right in stage (1), what actions will the Agent choose in stage (2), and what will be the parties' resulting total surplus?

(d) Can it ever be efficient to allocate the decision right to the Principal (even though the Principal takes no actions)? If so, why? If not, why not?

Problem 3: Holdup and Noise

At t = 1, a buyer makes a relationship-specific investment I, which pays off only if she is supplied with a widget by a seller at t = 3. Assuming the widget is supplied, the buyer's (gross) payoff from the investment is $I + \varepsilon$, where ε is a noise term uniformly distributed on [-x, x]. The cost of the investment is $I^2/2$, which is incurred by the buyer at t = 1. The buyer learns ε at t = 2. Neither I nor ε nor $I + \varepsilon$ is ever observed by the seller.

Assume that no contracts are feasible, and that the seller makes a take-it-or-leave-it offer to the buyer at t = 3. If the seller's offer is p then the payoffs are as follows: if the buyer accepts, $I + \varepsilon - p - I^2/2$ to the buyer and p to the seller; if the buyer rejects, $- I^2/2$ to the buyer and 0 to the seller. The parties are risk-neutral. There is no discounting.

The seller's optimal offer will be based on a conjecture about the buyer's investment, and this conjecture will be correct in equilibrium. For what values of x does this game have a pure-strategy equilibrium? For these values of x, compute the equilibrium investment by the buyer and compare it to the first-best investment. How (and why) does the buyer's investment vary with the support of the noise term, x?