PROBLEM SET 5: EX POST CONTROL
"DUE" FRIDAY OCTOBER 22.

Problem 1: Adaptation

Consider the following adaptation problem. There are two states ($s = s_1$ or s_2) that are equally likely, a binary decision ($d = 0$ or 1), and two parties (A and B). The payoffs are

$U_A(s_1, 0) = 0$, $U_A(s_1, 1) = 1$, $U_A(s_2, 0) = 0$, and $U_A(s_2, 1) = 4$;

$U_B(s_1, 0) = 6$, $U_B(s_1, 1) = 0$, $U_B(s_2, 0) = 2$, and $U_B(s_2, 1) = 0$.

The decision right is contractible ex ante, but the decision itself is not contractible even ex post, so the timing is: (1) bargain over the decision right, (2) A and B observe s, and (3) the owner of the decision right chooses d, which is observed by A and B.

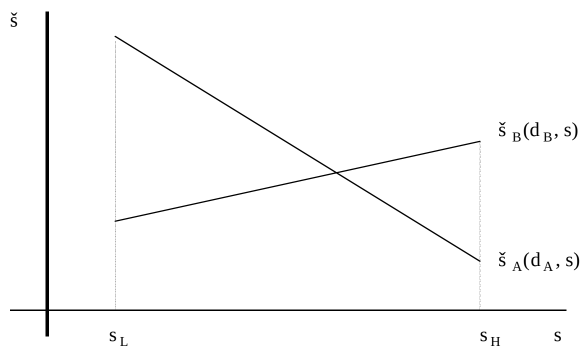
(For concreteness, suppose that party A owns the decision right at time 0, before the three stages defined above. Also, the bargaining in stage (1) implies the existence of side-payments; assume that the parties' utility functions are additively separable in money. Finally, the decision must be taken quickly once s is observed, so it is not possible to reallocate the decision right after the state is observed.)

a. What is the first-best decision rule?
b. What is the second-best allocation of the decision right?
c. Consider an infinitely repeated game based on the stage game above, but let side-payments be feasible throughout the game – specifically, in new stages (1.5), (2.5), and (3.5). Let the state be i.i.d. each period. Finally, suppose that A and B share the interest rate r per period. (i) Define trigger strategies that, if played, will produce the first-best outcome in every period. (ii) For what values of r are your trigger strategies a subgame-perfect Nash equilibrium of the repeated game? (iii) What initial allocation of the decision right (*i.e.*, in stage (1) of the first period of the repeated game) maximizes the largest value of r at which a trigger-strategy equilibrium can produce the first-best outcome in every period?

Problem 2: Contracting for Control

Two firms, A and B, are deciding who should control a decision right, D, that matters to both of them. The decision right is contractible, but the decision itself is not. The decision $d \in D$ is taken after the state $s \in S$ is publicly observed. Each firm then receives (non-contractible) private benefits $\pi_i(d, s)$. The firms have deep pockets and are risk-neutral.

There are only two possible decisions, d_A and d_B, in D. Regardless of the state, firm i always prefers decision d_i, with payoffs as follows: $\pi_i(d, s) = \sigma_i s + \rho_i > 0$ if $d = d_i$; $\pi_i(d, s) = 0$ if $d \neq d_i$. Assume that the state is uniformly distributed on $[s_L, s_H]$, and impose the following assumptions on σ_i and ρ_i, as shown in the figure below: $\sigma_A < 0$ and $\sigma_B > 0$; $0 < \sigma_B s_L + \rho_B < \sigma_A s_L + \rho_A$; and $0 < \sigma_A s_H + \rho_A < \sigma_B s_H + \rho_B$. (For some reason, the figure's notation mistakenly shows $s_i(d_i, s)$ instead of $\pi_i(d_i, s)$. Sorry!)



(a) What is the first-best decision rule, $d^{FB}(s)$, and what is the expected total surplus (denoted V^{FB}) generated by this decision rule?

(b) In the spot version of this model, which firm should control the decision right, and what is the expected total surplus (denoted V^{SP}) generated by such second-best spot control?

(c) Now consider the repeated-game version of this model, with interest rate r per period. Suppose that, in each period, there can be both "efficiency wage" payments, denoted by t, paid before the state is observed, and "discretionary bonus" payments, denoted by T(d, s), paid after the state and the decision are observed. Suppose that firm i begins the repeated game with control of the decision right. Define strategies that, if played, would produce the first-best in every period. For what values of r are your strategies a subgame-perfect Nash equilibrium of the repeated game? (Please choose strategies such that some values of r make your strategies subgame-perfect.)

(d) Given your answer to (c), which firm should begin the repeated game with control of the decision right, if we are interested in maximizing the set of values of r at which the first-best can be achieved?

(e) How would your answer to (d) change if the parameters changed so that $\sigma_B < 0$ (but we still had $\sigma_A < 0$, $0 < \sigma_B s_L + \rho_B < \sigma_A s_L + \rho_A$, and $0 < \sigma_A s_H + \rho_A < \sigma_B s_H + \rho_B$)?

Problem 3: Alliance

a) Consider the following one-shot "alliance" game. There are two players, firms A and B, who control decisions d_A and d_B, respectively, where d_i is chosen from the finite set D_i, where i = A, B. (Think of these decision rights as resulting from unmodeled assignments of assets and other control rights to the two firms. We now want to explore how this non-integrated governance structure will perform.)

There is a state of the world, s, drawn from the finite set S, according to the probability distribution p(s). The decisions and decision rights are not contractible, even after the state has been observed. Both firms have deep pockets, and so can make payments to each other, and the firms are risk-neutral. Thus, the timing of the one-shot game is as follows:

(1) s is publicly observed;
(2) d_A and d_B are chosen (simultaneously);
(3) firms A and B simultaneously choose payments m_A and m_B (both ≥ 0) to the other firm; and
(4) firm i receives the payoff $U_i(d_i, d_j, s) - m_i + m_j$

What conditions characterize a subgame-perfect Nash equilibrium of this one-shot alliance game?

b) To make this governance structure feel more like an alliance, suppose that the one-shot game is repeated forever, with the outcome (*i.e.*, realized state, chosen decisions, and payments) of each prior period publicly observed before the next period begins. Suppose both players share the discount rate r per period.

(i) Define the first-best decision rule. Define this as $d^{FB}(s)$.

Assume that the one-shot game has a unique subgame-perfect Nash equilibrium.

(ii) Define trigger strategies that, if played, would achieve the first-best $d^{FB}(s)$ in every period and that are a subgame-perfect Nash equilibrium of the repeated game for sufficiently high values of r. Determine the highest value of r at which your strategies are a subgame-perfect Nash equilibrium of the repeated game.

(iii) Now, suppose that payments are illegal, so $m_A = m_B = 0$. Define trigger strategies that, if played, would achieve the first-best $d^{FB}(s)$ in every period and that are a subgame-perfect Nash equilibrium of the repeated game for sufficiently high values of r. Determine the highest value of r at which your strategies are a subgame-perfect Nash equilibrium of the repeated game.