Problem Set 5
14.281, Fall 2004
Due date: Friday, Nov 19 at 4:30 pm (before make-up class).

Problem 1. (Renegotiation with Cross Investments, Guriev 2003)

Consider a bilateral trade setting with a buyer B and a seller S. There is one unit of good that belongs to the seller at time t=0. At t=1/2, the buyer and the seller simultaneously make investment $\beta \geq 0$ and $\sigma \geq 0$ respectively. At time t=1, both parties observe the state $\omega \in \Omega$, the seller's production cost $c(\omega,\beta,\sigma)$, and the buyer's valuation of the good $v(\omega,\beta,\sigma)$. Once the state is observed, the parties renegotiate whether to trade and at what price. The questions below examine several renegotiation processes. We assume that the trade is always efficient, so $c(\omega,\beta,\sigma)<v(\omega,\beta,\sigma)$ for all parameters. The p.d.f. of $\omega$, f( $\omega$ ) is common knowledge. We define $B(\beta,\sigma)=E[v(\omega,\beta,\sigma)]$, $C(\beta,\sigma)=E[c(\omega,\beta,\sigma)]$. We assume that B is strictly concave and C is strictly convex. For now, we focus on the interior solutions of the problem.

a. (No Contract) Suppose no contracts are written at t=0. At t=1, the parties observe the state of nature and decide the division of surplus through Nash Bargaining. Assume that both parties have equal bargaining powers, what will the initial investment levels be? How do they compare to first-best levels?

b. (Fixed-Price Contract) Suppose at t=0 the parties can sign a contract to trade at price $\overline{p}$. When will the parties renegotiate at t=1? What will be the investment levels at t=0? Show that the fixed-price contract is efficient if and only if $B^*_\sigma = S^*_\beta = 0$. Furthermore, show that the fixed-price contract is more efficient if $B_\beta > S_\beta > 0$, and $S_\sigma > B_\sigma > 0$. When will no contract at date t = 0 be better than a fixed-price contract?

c. (Option Contracts) Consider the following contract: At time t=1 the seller gets a right to sell the good to the buyer at $p^s$ even if the buyer does not want to take it. Meanwhile, the buyer gets a right to oblige the seller to deliver the good at time t=1 at price $p^b > p^s$. Assume Nash Bargaining with equal bargaining power at time t=1. Then what's the equilibrium payoff of the buyer and the seller with investment $\beta,\sigma$? If the shock is multiplicative, i.e. $c(\omega,\beta,\sigma)=\omega\overline{c}(\beta,\sigma)$, $v(\omega,\beta,\sigma)=\omega\overline{v}(\beta,\sigma)$, show that there exists $p^b*, p^s*$ that will induce the first best level of investment $\beta*$, $\sigma*$ at t=1/2.

d. Extra Credit: Will the first best level of investment be reached if the shock has a more general structure?

Problem 2.

Consider the following three period health insurance problem. The insured has a (certain) income y in each period $t = 1,2,3$. The periodic utility function is $u(c_t)$. There is no saving (other than through an insurance contract). In period 2 and period 3 there is a chance of ill health. Let $h_t$ be the realized health costs in period t, $t = 2,3$. Assume $h_3 = h_2 + \varepsilon$, where $\varepsilon$ has expectation zero and positive variance. An insurance contract specifies (a) in period 1 an initial premium; (b) in period 2 a reimbursement (possibly partial) for the health costs $h_2$ and (unless the insured stops the policy) a premium also contingent on $h_2$; and (c) in period 3 a reimbursement for health costs $h_3$ (possibly contingent on $h_2$ as well). The industry is competitive (zero profits from contracts) and all insurance companies see the same information. Thus, as in Hendel and Lizzeri, the insured will quit in period 2 if she can get a better contract offer from another firm.

a. Set up a program that solves for the optimal long-term (renegotiation proof) contract.

b. Show that the optimal contract entails at least as high a level of consumption in period 3 as in period 2 and in period 2 as in period 1.

c. Compared with a model with just two periods, will the initial premium in the three period model be higher or lower?

Problem 3.

Consider a two period ($t = 1,2$) repeated principal agent model with two levels of output ($x_t$ = S or F) each period. Let the agent's market opportunity *after* period t (the expected utility the agent can get by switching to another principal) be $z_t$. This market opportunity may depend (possibly stochastically) on the agent's action in that period $a_t$. The agent cannot access capital markets; the earnings from the period must either be consumed or used (in part or wholly) for an up-front payment to the principal (a bond for the next period). The principal is risk neutral, the agent is risk averse with utility function $\Sigma_t (u(c_t) - c(a_t))$. The principal's reservation utility over the two periods is assumed to be zero.

a. Assume first that output $x_t$ is contractible and that $z_t$ is a function of the agent's action only through $x_t$. Show that in this case, the optimal long-term contract can be decomposed into a sequence of (two) short-term contracts with the principal earning an expected payoff of zero at the beginning of the first period as well as after each of the two outcomes S and F in period 1.

b. Suppose now that the agent's action can influence $z_t$ through the unobserved choice $a_t$. Provide an example that shows that in this case it may not be possible to decompose the optimal long-term contract into a sequence of short-term contracts. (Hint: you may want to use a multi-task model, with one of the agent's actions influencing output and the other influencing the outside option.)