Problem Set 6
14.821, Fall 2004
Due date: Monday, Dec 6 (before class)

Problem 1.

A project that costs I dollars pays off either R (with probability p) or 0 (with probability 1-p). The project has positive net present value: pR - I > 0. Suppose such a project is available to a firm in each of two periods. The first-period project is stochastically independent of the second-period project, but otherwise identical. The firm has initial cash in the amount A < I. To invest the firm must therefore obtain the balance I-A from an investor. Investors demand zero interest and are willing to enter into any contract that guarantees them non-negative profits. If project outcomes could be verified, it would be easy to write a contract that made it possible to finance each of the two projects. Unfortunately, outcomes are not verifiable. The only kind of contract that the investor can use to induce the firm to repay the initial loan is to promise the firm financing of the second period project if the firm repays the investor the amount x after the first period.

a. Write down the minimal amount of internal cash A necessary to obtain financing for a project in the first period. (There may be several cases to consider as R and p vary.)

b. Suppose, instead of assuming that the investor can commit to financing the second period project contingent on a repayment x, that the investor can observe the first period outcome and contingent on this observation let the firm invest in the second period (ie the investor has control rights over the decision to continue). Suppose further that the investor makes a take-it-or-leave-it offer to the firm to continue, which takes the form of letting the firm continue if it repays x. In this new situation, will the firm need more or less or the same amount of initial cash to be able to go ahead with the first period project.

Note: the firm can ask the investor to pay more than I-A in the initial period.

Problem 2: On Groves' mechanism.

A seller may supply a single object to a buyer. Let x be 1 if trade takes place and 0 if not. Let t_S be the amount of money that the seller *receives* and t_B the amount that the buyer *pays*. Let v be the buyer's valuation and s be the seller's valuation of the object and assume preferences are quasi-linear. We can then normalize utilities so that the seller's utility is $t_S - cx$ and the buyer's utility is $vx - t_B$. Assume the preference parameters v and c are independently drawn from a uniform distribution on [0,1]. The buyer knows v and the seller knows c.

a. What is the efficient rule for trade?

b. Let m_S and m_B be the seller's respectively the buyer's reported preference for the object. Determine the set of direct mechanisms (expressed as a function of the reports) that admit truth telling as a dominant strategy and implement efficient trade (ie. the set of Groves mechanisms).

c. Show that there is a unique Groves mechanism that has the property that whenever trade does not occur, the transfer payments are set equal to zero ($t_B = t_S = 0$). Is this mechanism feasible?

d. Show that there is no Groves mechanism for which the budget breaks even for all reported preferences. (Do *not* invoke Myerson-Satterthwaite's theorem, but rather argue the case directly.)

Problem 3.

A seller owns one unit of a good which she values at c. (The value c can be thought of as the quality of the good.) A buyer may buy the unit from the seller. The seller's valuation is equal to c_1 or to c_2 with equal probabilities. The buyers valuation for the good is v_1 if $c=c_2$ and v_2 if $c=c_1$, where $v_1 > c_1$ and $v_2 > c_2$. The buyer thus has no private information. Assume that $\frac{1}{2}(v_1 + v_2) < c_2$.

a. Show that efficiency is inconsistent with the seller's and the buyer's individual rationality and incentive compatibility.

b. Give two reasons why you cannot appeal to the Myerson-Satterthwaite result in this question.

Problem 4.

An entrepreneur runs a company that will generate profits $x = e + \varepsilon$, where ε is a normally distributed random term with mean zero and variance σ^2 and e is the entrepreneur's effort. The entrepreneur's private cost of effort is c(e). The agent has exponential utility with risk aversion coefficient r.

Two insurance companies are independently planning to take an equity stake in the entrepreneurial firm. Formally, they are offering to share part of the firm's profits using linear incentive schemes $s_i(x) = \alpha_i x + \beta_i$, where i = 1,2 indexes the two firms.

a. Assuming that the insurance companies make independent, unobserved offers $s_i(x)$ to the entrepreneur, what offers will be made in equilibrium? How does this equilibrium compare with a second-best incentive design (a single insurance company financing the entrepreneur)?

b. Assuming that company 1 makes an offer first, followed by an offer by company 2, what equilibria will arise in this Stackelberg set-up?