RELATIONAL TEAM INCENTIVES AND OWNERSHIP

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Abstract. The firm is modeled as a team experiencing moral hazard that meets repeatedly. Effort incentives derive from both court-enforced ownership shares over the stream of profits, and self-enforced performance bonuses contingent on non-verifiable performance measures, honored through the threat of future punishments. The optimal arrangement is determined by the degree of moral hazard within the team. When efforts are observable, ownership shares are dispersed so that no single player is the full residual claimant of output, and every player receives bonus incentives. When performance measures are sufficiently noisy, ownership is concentrated in the hands of a single player to the degree that she receives no additional incentives. Moreover, this player can be charged with the task of paying all the performance bonuses of her peers, and is thus viewed as an endogenously chosen principal.

1. Introduction

In modern economies most human resources are allocated within firms, where efforts from multiple parties are combined towards the production of joint output. Such team arrangements exploit complementarities, but face the important problem of having to share output among the members of the firm. The resulting externalities lead to an incentive problem that in general cannot be resolved using ownership shares alone –Alchian and Demsetz [1972], Holmstrom [1982].

Joint production, however, is associated to close interaction, implying that firm members will typically possess information about each others' efforts that is substantially more accurate than the information contained in joint output alone. This information will tend to be "soft", i.e., too costly to be verified by courts, and thus cannot be the basis for court-enforced incentive contracts. But the interaction among firm members will also typically occur within a repeated relationship, allowing for self-enforced contracts based on this additional information. Such implicit agreements

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will be enforced through the threat of future punishments using the relationship, e.g., the threat of separation, and will be effective insofar as this relationship creates a sufficiently large future surplus, or *relational capital*, which is at stake when the agreements are reneged upon.[1]

This paper takes the position that implicit contracts are a central source of effort incentives, and studies how the firm will organize to best exploit their potential. I model the firm as a team of players who collaborate repeatedly, and have two sources of incentives: *ownership shares* and *money transfers*. Ownership shares will simply be court-enforced property rights over the stream of profits, assumed to be verifiable. Money transfers will take the form of self-enforced performance bonuses based on informative performance measures that are correlated to effort, but not verifiable. These bonuses will be equivalent to arbitrary continuation rewards based on performance (such as salary raises and severance), and are adopted for concreteness.

Both incentive sources will be limited. Ownership shares cannot exceed 100% of the firm. Performance bonuses, on the other hand, will only be honored if the party required to make the payment has sufficient future surplus to lose upon reneging. But total surplus is assumed to be scarce, limiting the maximum size of these payments together with their incentive potential. I study the problem of organizational design along these two dimensions, encompassing both the allocation of ownership shares, and the implicit agreement to carry out performance bonuses, in a way that best promotes effort.

The literature on incentives in organizations is centered around the principal-agent paradigm, which focuses on optimal agent compensation while taking as given the agency relationship itself.[2] In particular, there is a pre-determined principal who is the full residual claimant of output, compensates the agent (or agents), and does not directly contribute to output. In practice, however, residual claimants will also commonly participate in production –as in owner-managed firms– and residual claim need not be fully allocated to a single party –as in professional partnerships. In other words, the agency relationship represents an organizational choice variable as well, the importance of which is suggested by the patterns of ownership across existing firms –Hansmann [1996].

[1]The widespread use of implicit contracts is reviewed in Levin [2003]. Examples range from direct bonus payments based on performance to promotions to efficiency wages.

[2]See Gibbons [1998] for a review. A central theme has been the allocation of risk, from which I abstract below.

This paper endogenizes the agency relationship –along the dimensions of residual claim and performance pay– by combining the agency problem with a team problem, where effort externalities flow in every direction. The building blocks are the static team model of Holmstrom [1982], now studied in a repeated environment, and the literature on repeated agency with implicit contracts (e.g., Bull [1987], Spear and Srivastava [1987], MacLeod and Malcomson [1989], Baker, Gibbons and Murphy [1994], Schmidt and Schnitzer [1994], Pearce and Stacchetti [1998], Che and Yoo [2001], Levin [2003]), which is extended to allow for endogenous agency.[3] The model is closest to Levin's [2003] repeated agency, and indeed capitalizes on many of his results (see also Levin [1998] and Dewhurst [2000]). The main distinction is again the fact that ownership is endogenous and efforts are multi-sided. As a result, the analysis shifts from the study of optimal agent compensation to the broader theme of organizational design.

Also related is the literature on ownership and integration spawned by Grossman and Hart [1986], and Hart and Moore [1990]. They focus on the complementary notion of ownership, namely, the possession of residual control rights over assets, which plays a key role in providing incentives for ex-ante investments. Their theory is designed to study the boundaries of the firm, rather than its internal organization, and thus serves a complementary purpose. In this line of inquiry, Baker, Gibbons and Murphy [2002, 2003] view the interplay between explicit and implicit contracts as key to understanding integration and strategic alliances. Their work also stresses the issue of allocating discretion (and trust) across players.[4]

The gist of the analysis below is the marginal rate at which implicit incentives (performance bonuses) and explicit incentives (ownership shares) can be traded off while inducing the same levels of effort. This incentive trade-off represents the marginal returns of ownership in terms of reducing the need of (scarce) implicit incentives for any given player, and therefore determines which player has a comparative advantage towards receiving ownership over bonus payments. The incentive trade-off will favor players that are both highly productive and hard to monitor by their peers, in the sense that their contribution is the hardest to assess. These features imply that ownership shares are especially effective towards providing incentives, while bonus payments are especially weak.

[3] Implicit contracts, and their interaction with explicit contracts, have also been widely studied outside agency relationships, e.g., Klein and Leffler [1981], Bernheim and Whinston [1998].

[4] See also Williamson [1985], Garvey [1995], Bragelien [1998], and Halonen [2001]. In Levin and Rayo [2003] we study the optimal degree of control-right concentration under relational contracting.

The main result is a comparative static concerning the impact that the exogenous degree of observability of peer efforts has over the allocation of ownership shares and implicit incentives. In particular, I consider environments differing in how informative (or noisy) the available performance measures are. When efforts can be perfectly observed (section 3) it will be optimal to disperse ownership shares so that every player receives implicit incentives. Such dispersion is caused by a form of decreasing returns to ownership, associated to the fact that additional shares in hands of a given player reduce the size of her most tempting effort deviations, and the smaller a deviation the easier it is to deter using bonus payments instead. Consequently, ownership will have a diminishing marginal impact in terms of reducing the need for implicit incentives, and as a result it will never be fully concentrated.

Under hidden efforts, in contrast, even small deviations will be costly to deter using implicit incentives as they are the most difficult to detect, implying that the marginal impact of increasing ownership shares will always remain high (section 4). In fact, when the performance measures are sufficiently noisy so that "local" (or infinitesimal) deviations become the most tempting, returns to ownership never decrease. It is then optimal to adopt a corner solution that concentrates ownership in hands of a single player with the highest returns to the point where she receives no implicit incentives. In the extreme of first-best efforts this implies full concentration of ownership. Moreover, this player can always be charged with the task of paying all the performance bonuses of her peers. In this sense she becomes an endogenously chosen principal. Section 5 extends the analysis to intermediate information scenarios, leading to intermediate results. Section 6 discusses some related empirical patterns of ownership.

2. Model

N risk-neutral players participate in joint production. The are two stages. In the first, every player i simultaneously selects an effort level $e_i \in \mathbb{R}_+$ at a private cost $c_i(e_i)$, which is smooth, strictly increasing and strictly convex. The vector of efforts e stochastically determines joint output x (i.e., profits) to be divided across players, with an expected value $E[x \mid e]$ that is smooth, strictly increasing, and concave in e. Each effort is private information, but produces a noisy performance measure y_i commonly observed by all players (but not verifiable by courts), which delivers valuable information about e_i. I assume in fact that joint output provides no information about e_i beyond that contained in y_i. In the second stage, players transfer money to each other contingent on the performance vector y, and payoffs are realized. Let $\tau_i(y)$ denote the net contingent payment received by i, with $\sum_i \tau_i(y) = 0$.[5]

Players will ex-ante agree on a court-enforced contract that specifies both the division of joint output, verifiable by courts, and the payment of non-contingent wages among themselves. Output will be divided according to *ownership shares* $\alpha_i \in [0, 1]$, representing the share of output that accrues to each player, with $\sum_i \alpha_i = 1$. Non-contingent wages, on the other hand, are simply money transfers that do not depend on y (as it is not verifiable). Let ω_i denote the net wage received by i, with $\sum_i \omega_i = 0$.[6]

The payoff for player i is

$$\alpha_i x + \omega_i + \tau_i(y) - c_i(e_i).$$

Wages ω_i will not constitute a source of effort incentives. Rather, they are used only to transfer surplus across players (and induce participation). Since the focus will be on contracts that maximize joint surplus, in what follows these wages will be sent to the background.

Transfers $\tau_i(y)$ are voluntary, and will only be honored through the threat of future punishments. Indeed, I assume that players are involved in a repeated relationship (outside the model), and surplus is created by virtue of their interaction. Separation can then be used as a trigger punishment device whenever a voluntary payment is reneged upon. In particular, suppose that after the game described above is over, a reneging player i can be excluded from future interaction, leading her to receive a constant continuation value $\overline{u}_i$, which represents a reservation payoff. If no player reneges, on the other hand, joint production continues, leading to a continuation payoff of u_i for each player. Voluntary future participation will require $u_i \geq \overline{u}_i$. (Subsets of players might continue collaborating after reneging takes place, but such continuation paths will not affect the analysis provided they do not alter the reservation payoff for the reneging player.)

A player will be tempted to renege on a voluntary transfer $\tau_i(y)$ whenever it is negative, meaning she has to make a net payment to her peers. When deciding

[5] Money burning $\sum_i \tau_i(y) < 0$ could potentially be used as a collective punishment device, but will not be optimal in the present context. On the other hand, conditioning transfers on x as well as y would only introduce dispersion that makes these transfers harder to enforce.

Possible extensions of the model include both endogenizing the quality of the performance measures through costly monitoring (Alchian and Demsetz [1972]), and allowing for subjective assessments of performance (Levin [2003], McLeod [2003]).

[6] Court-enforced contracts are thus assumed to be linear functions of output: $\alpha_i x + \omega_i$. The study of non-linear rules, such as stock options, is left for future work.

whether or not to do so, she will compare her continuation payoffs for each alternative. If she does not renege, her continuation payoff is $\tau_i(y) + u_i$, while reneging leads to $\overline{u}_i$. Thus, players will honor their voluntary transfers if and only if constraint (T) is satisfied:

$$(T) \qquad -\tau_i(y) \leq u_i - \overline{u}_i \text{ for all } y \text{ and all } i.$$

A large value for $u_i - \overline{u}_i$ will increase the maximum payment that can be required from player i when her peers exhibit good performance, thus enhancing effort incentives.

The vector of continuation values u will be a choice variable in the model. In particular, letting U denote the (fixed) total discounted future surplus achieved when players continue with joint production, I assume that u can take any value subject to $\sum_i u_i = U$, and $u_i \geq \overline{u}_i$. As in Levin [2003], such redistributions of surplus simply require that players can transfer money among themselves in future periods (assuming a common discount factor). This would precisely be the role played by court-enforced wages in subsequent stages of production.

A. Relational Contracts

On top of the court-enforced output shares α, players will ex-ante agree on a self-enforced contract, consisting of a prescribed vector of efforts e, a vector of promised voluntary payments $\tau(\cdot)$, which without loss of generality is assumed to satisfy (T), and a vector of continuation values u with the restrictions noted above. The problem facing the team is to select a *relational contract* $\langle \alpha, e, \tau(\cdot), u \rangle$ (specifying all aspects of the relationship) that maximizes expected joint surplus subject to the corresponding incentive, future participation, and feasibility constraints:

$$\text{(I)} \qquad \max_{\langle \alpha, e, \tau(\cdot), u \rangle} E\left[x \mid e\right] - \sum_i c_i(e_i)$$

subject to, for all i:

$$(IC) \qquad E\left[\alpha_i x + \tau_i(y) \mid e\right] - c_i(e_i) \geq E\left[\alpha_i x + \tau_i(y) \mid e_i', e_{-i}\right] - c_i(e_i') \text{ for all } e_i',$$

$$(T) \qquad -\tau_i(y) \leq u_i - \overline{u}_i \text{ for all } y,$$

$$(U) \qquad u_i \geq \overline{u}_i, \text{ and } \sum_i u_i = U.$$

The first constraint (IC) is the effort incentive constraint: Given that each player i anticipates voluntary transfers to be honored, and given that her opponents follow the prescribed efforts e_{-i}, (IC) requires that the prescribed effort e_i is preferred over any alternative e_i'. The second constraint (T), as noted above, guarantees that transfers will actually be honored. The last two constraints (U) correspond to the limits on how continuation values can be assigned.[7]

An important feature of this problem is that the continuation values u do not depend on performance y. Alternatively, players could agree on a self-enforcing contract that rewards performance using changes in continuation values instead of money transfers (or any combination of the two). However, under risk-neutrality these two instruments are perfect substitutes in all the constraints and payoffs. As a consequence, any contract such that u depends on y can be replaced with a payoff-equivalent contract with a constant u. Intuitively, in such a contract all performance rewards are "settled" on the spot using immediate money transfers, instead of being carried out through future transfers (or other actions). I focus on this type of contracts for concreteness, but exactly the same results would hold under the more general class of rewards.

I will impose a key simplification over the shape of implicit incentives. I assume that the performance measures y_i are independent from each other (conditional on effort),[8] and that voluntary transfers $\tau_i(y)$ take an additively separable form:

$$\tau_i(y) = b_i(y_i) + v_i(y_{-i}).$$

That is, $\tau_i(y)$ will only depend on y_i through the function $b_i(\cdot)$, a *performance bonus*, which in turn does not depend on other players' signals y_{-i}. This bonus will be the only source of effort incentives. The second function $v_i(y_{-i})$ will simply be used to balance the budget across players.[9] These assumptions will allow for a highly tractable problem.

In the present setting, the cost of imposing additive separability is that it will rule out winner-take-all tournaments where, for each realization of y, the winner receives

[7]These constraints define a perfect public equilibrium (adopted here without loss), see Fudenberg, Levine and Maskin [1994]. The recursive approach follows Abreu, Pearce and Stacchetti [1990].

In Rayo [2002] I develop an infinite horizon version of the present model, where the above game is repeated indefinitely and U becomes endogenous. There I show that *stationary* contracts are optimal, where, on the path of play, the same court enforced contracts $\langle \alpha, \omega \rangle$ are adopted every period, and the same actions $\langle e, \tau(\cdot) \rangle$ are prescribed. The optimal stationary contract will precisely be the one that solves problem (I), taking as given the maximum possible continuation surplus U. These derivations are a straightforward extension of Levin [2003], and are omitted here for brevity.

[8]Consider, for example, a team of computer engineers working on the same software package. Each one might work on a separate but complementary aspect of the program, while observing the independent quality of the code written by her peers (although not necessarily the effort itself).

[9]In contracts where u depends on y, separability would need to be imposed over $\tau + u$.

a continuation reward $\tau_i(y) + u_i$ equal to $\overline{u}_i$ plus all surplus $U - \sum_i \overline{u}_i$, while every other player receives only her reservation value. Although such tournaments will have no statistical advantage over separable transfers (due to the independence of each y_i, Mookherjee [1984]), they can potentially relax the reneging constraints (T), and thus become optimal. (Other forms of non-separable transfers will be dominated by these tournaments.)[10]

These extreme tournaments, however, would present a number of practical disadvantages (Baron and Kreps [1999], Bandiera, Barankay and Rasul [2003]). They can easily hinder cooperation, reduce the incentives or "moral" of players falling behind, and even lead to sabotage and collusion against high effort. They also require a fine-tuned handicapping rule to determine a single winner among all the members of a firm, especially problematic when the firm is large and tasks are heterogeneous.

In practice, some types of tournaments are certainly frequent, particularly when it comes to promotions into new jobs where there are fewer spots than participants and there is learning involved. These tournaments, however, occur only among subgroups of employees, and tend to be avoided among individuals directly collaborating with each other (Lazear [1998], and Che and Yoo [2001]). In other words, at least some degree of separability seems to be the norm, more so across employees of known ability working on tasks that can be independently assessed.

B. Preliminaries

Once additively separable transfers are adopted, all voluntary transfer constraints (T) can be reduced to a single inequality, which will bound the total discretion arising from these transfers. Let $\overline{U} := \sum_i \overline{u}_i$, so that $U - \overline{U}$ corresponds to the net future

[10] Winner-take-all tournaments correspond to the optimal "bang-bang" extreme continuation values in Abreu, Pearce and Stacchetti [1990]. See also Levin [1998], and Dewhurst [2000]. In this case, they place the highest possible reward over the highest weighted realization of y_i across players, and they call for money burning when every player exhibits low performance. (The analysis becomes problematic when calculating these weights.) In terms of the optimal ownership structure under tournaments, when efforts are unobservable, dispersing ownership will have the advantage of allowing everyone to participate in the tournament. However, if there is a player who is relatively important for output, and whose performance is relatively hard to assess by her peers, increasing ownership in her hands will also be advantageous (as in section 4). This second motive can easily dominate and again lead all the way to fully concentrated ownership (as hidden efforts imply that marginal returns to ownership will always remain high). Under observable efforts, on the other hand, tournaments are of no use.

surplus available to the team. (I assume throughout that $U-\overline{U}>0$.) Also let $\Delta b_i := \sup_{y_i} b(y_i) - \inf_{y_i} b(y_i)$, called the *power* of player i's implicit incentives. Associated to such power, a level of discretion must be given to the players ultimately in charge of paying the bonus through the budget-balancing functions $v(\cdot)$. Discretion, in turn, will only be exercised against short-run opportunism to the extent that such players face a threat of losing sufficient future surplus. Thus, the larger the bonus power, the higher the fraction of net surplus $U-\overline{U}$ required to enforce it.

Formally, as shown in lemma 1, a transfer agreement can be enforced if and only if (DE) (for "dynamic enforcement") is satisfied:

$$(DE) \qquad \sum_i \Delta b_i \le U - \overline{U}.$$

Future surplus, $U-\overline{U}$, called *relational capital,* will be the source of implicit incentives, while the combined bonus power will be their cost. (DE) can thus be viewed as an implicit-incentive feasibility constraint.[11]

Lemma 1. *Under additively separable voluntary transfers, in problem* (I) *constraint* (DE) *is equivalent to* (T).

Proof. I begin with $(T) \Rightarrow (DE)$. (T) implies that $-\sum_i \inf_y \tau_i(y) \le U - \overline{U}$. Balanced transfers, on the other hand, imply that $\sum_i b_i(y_i) = -\sum_i v_i(y_{-i})$ for all y, and therefore $\sup_y \sum_i b_i(y_i) = \sum_i \sup_{y_i} b_i(y_i) = -\inf_y \sum_i v_i(y_{-i})$. Combining these facts we obtain

$$\begin{aligned}\sum_i \Delta b_i &= \sum_i \left\{ \sup_{y_i} b_i(y_i) - \inf_{y_i} b_i(y_i) \right\} = -\inf_y \sum_i v_i(y_{-i}) - \sum_i \inf_{y_i} b_i(y_i) \\ &\le -\sum_i \left\{ \inf_{y_{-i}} v_i(y_{-i}) + \inf_{y_i} b_i(y_i) \right\} = -\sum_i \inf_y \tau_i(y) \le U - \overline{U}.\end{aligned}$$

For $(DE) \Rightarrow (T)$, it suffices to show that whenever (DE) holds there exist functions $v_i(\cdot)$ that balance the budget, and feasible continuation values u (satisfying constraint (U)) such that (T) holds. Notice that the latter simply requires $-\sum_i \inf_y \tau_i(y) \le U - \overline{U}$. Let the payment of each $b_i(\cdot)$ be divided equally among the remaining players

[11] In practice courts could also enforce contracts that affect individual reservation payoffs (e.g., severance payments and bonding). However, from (DE) we see that such arrangements would not change the analysis provided they only represent transfers across players without any surplus destruction, so that the *sum* of reservation values $\overline{U}$ is not altered (and provided these reservation values are not contingent on y).

(any other linear rule would also work): $v_i(y_{-i}) = -\frac{1}{N-1}\sum_{j\neq i} b_j(y_j)$, which in fact balances the budget and implies

$$-\sum_i \inf_y \tau_i(y) = -\sum_i \inf_{y_i} b_i(y_i) + \frac{1}{N-1}\sum_i \sum_{j\neq i} \sup_{y_j} b_j(y_j) = \sum_i \Delta b_i \leq U - \overline{U}.$$

∎

When constraint (T) is replaced with (DE), both the continuation values u and the budget-balancing functions $v(\cdot)$ can be sent to the background. The reason is that they have no impact over total surplus, (IC), or (DE). Notice also that constraints (U) no longer play a role.[12] Problem (I) will thus simplify to that of selecting a reduced relational contract $\langle \alpha, e, b(\cdot)\rangle$ subject only to (IC) and (DE).

The next step is to merge constraints (IC) and (DE) into a single inequality, called $(IC\text{-}DE)$. This is done by expressing bonus power Δb_i as a function of α_i and e, in a way that captures the restrictions imposed by (IC). Once the two constraints are merged, problem (I) will reduce to one of selecting only α and e:

(II)
$$\max_{\langle \alpha, e\rangle} E\left[x \mid e\right] - \sum_i c_i(e_i)$$

subject to:

$(IC\text{-}DE)$
$$\sum_i \Delta b_i(\alpha_i, e) \leq U - \overline{U}.$$

$\Delta b_i(\alpha_i, e)$ represents the cost, in terms of relational capital, of inducing player i to select her prescribed effort for each level of α_i. Since bonus power and ownership shares are incentive substitutes, we have $\partial \Delta b_i/\partial \alpha_i < 0$. The absolute value of this derivative, $-\partial \Delta b_i/\partial \alpha_i$, called the *incentive trade-off*, will indicate the rate at which implicit and explicit incentives can be substituted while inducing the same level of effort.

Ownership only enters problem (II) through $(IC\text{-}DE)$. Consequently, it will be allocated with the sole purpose of relaxing this constraint. Indeed, the analysis will be centered around the dual problem of selecting α to minimize $\sum_i \Delta b_i(\alpha_i, e)$, as dictated by the shapes of $-\partial \Delta b_i/\partial \alpha_i$. In this dual problem, the incentive trade-offs will represent the marginal savings in relational capital when increasing ownership

[12] The only requirement for u and $v(\cdot)$ is that, whenever a player is given the task of paying a fraction of her peers' bonuses (through $v_i(\cdot)$), she must also be allocated a fraction of U large enough to enforce these payments. But under (DE) such an arrangement always exists. I return to this point in section 6.

in hands of a given player. The marginal ownership share will then be given to the player with the most attractive (i.e., highest) incentive trade-off.

3. Observable Efforts

Suppose $y \equiv e$, so that all effort levels are fully observed by the team. I first derive constraint (*IC-DE*), and then address the dual problem of minimizing its left-hand side $\sum_i \Delta b_i$. Central to the analysis will be the effort levels that would be selected absent any implicit incentives: Let the *static best response* for player i, denoted by $e_i^s(\alpha_i, e_{-i})$, be her optimal effort when facing $\Delta b_i = 0$, given her ownership share α_i (the only source of incentives), and peer efforts e_{-i}:

$$e_i^s(\alpha_i, e_{-i}) := \arg\max_{e_i'} \left\{\alpha_i E\left[x \mid e_i', e_{-i}\right] - c_i(e_i')\right\}.$$

I assume e_i^s is bounded, and positive for all $\alpha_i > 0$.

The purpose of implicit incentives will be precisely to implement efforts higher than e_i^s. In order to achieve this, each player must receive a bonus that at least deters a deviation from the prescribed effort e_i to e_i^s. From i's (IC) constraint, this requires that her bonus (now a function of effort) satisfies

$$b_i(e_i) - b_i(e_i^s) \geq \alpha_i E\left[x \mid e_i^s, e_{-i}\right] - c_i(e_i^s) - \left\{\alpha_i E\left[x \mid e\right] - c_i(e_i)\right\}.$$

The right-hand side represents the short run gain from performing the deviation to e_i^s. Denote this gain by $\varphi_i(\alpha_i, e)$, which imposes a lower bound for the bonus power Δb_i required to implement e_i. But once this static deviation is deterred, any other deviation can be deterred at no additional bonus cost. For example, setting $b_i(e_i') = b_i(e_i^s)$, for every deviation $e_i' \neq e_i$, is sufficient to satisfy (IC). Formally, only the "global" incentive constraint corresponding to e_i^s will bind. As a consequence, the minimum bonus power required to implement e_i is exactly $\varphi_i(\alpha_i, e)$:

Proposition 1. *When efforts are observable, effort levels e can be implemented under ownership structure α if and only if*

$$(IC\text{-}DE) \qquad \sum_i \Delta b_i(\alpha_i, e) = \sum_i \varphi_i(\alpha_i, e) \leq U - \overline{U}.$$

The final step is to attack problem (II) through the dual problem of selecting α as to minimize the left-hand side of (*IC-DE*), given the desired efforts. Suppose we knew the optimal levels of efforts e^*, i.e., those that solve (II). Then, any α that minimizes $\sum_i \Delta b_i(\alpha_i, e^*)$, called an *efficient* ownership structure, must also be optimal –recall that α only enters problem (II) through the constraint. I focus on characterizing these efficient levels of α, for which only partial knowledge of e^* will suffice.

Although every efficient α will be optimal as well, the converse need not be true. This will depend on whether or not constraint (IC-DE) binds. When it does bind, i.e., $U-\overline{U}$ is scarce, any optimal α must also be efficient, and therefore the dual approach will characterize all optimal ownership structures. But, as $U-\overline{U}$ grows, (IC-DE) will eventually become slacked, and the first-best levels of efforts e^{FB} (maximizing total surplus) can also be implemented under somewhat inefficient ownership structures. The advantage of an efficient α is that it will be optimal for all values of $U-\overline{U}$ that allow for the first best.

The efficient allocation of ownership is determined by the incentive trade-offs $-\partial \Delta b_i/\partial \alpha_i|_{e^*} = -\partial \varphi_i(\alpha_i, e^*)/\partial \alpha_i$, given optimal efforts e^*. Application of the envelope theorem yields

$$-\frac{\partial}{\partial \alpha_i}\varphi_i(\alpha_i, e^*) = E\left[x \mid e_i^*, e_{-i}^*\right] - E\left[x \mid e_i^s, e_{-i}^*\right]. \tag{1}$$

This trade-off is positive whenever $e_i^* > e_i^s$, and *decreasing* in α_i since e_i^s increases with α_i. In other words, the marginal returns to ownership (in the dual problem) are positive as long as a player is prescribed an effort higher than her static choice, but are decreasing and converge to zero as $e_i^s \to e_i^*$. Intuitively, the marginal impact of the first units of α_i is high because they help deter large deviations, which are expensive to deter using bonuses instead, while additional units of α_i will only deter smaller and cheaper deviations. In the extreme where e_i^s approaches e_i^* additional shares are of virtually no use. Such decreasing returns will imply that ownership, as well as implicit incentives (their counterpart), should be dispersed:

Theorem 1. *When efforts are observable, any efficient ownership structure is such that:*

(a) *ownership is non-extreme ($\alpha_i < 1$ for all i), and*

(b) *all players exerting positive effort receive implicit incentives ($\Delta b_i > 0$).*

Proof. Suppose contract $\langle \alpha^*, e^* \rangle$ is optimal and α^* is efficient. The following properties are a straightforward consequence of the optimality of $\langle \alpha^*, e^* \rangle$, and are left to the reader: (i) $e_i^* \geq e_i^s$ for all i; (ii) $\Delta b_j(\alpha_j^*, e^*) > 0$ for some j, and therefore $e_j^* > e_j^s$ (since either $e^* = e^{FB}$ or (IC-DE) binds); and (iii) $\alpha_i^* = 1$ implies $e_i^* = e_i^s$ (since i fully internalizes the benefits of her effort).

The proof hinges on the following necessary condition for efficiency of α^*:

$$\Delta b_i(\alpha_i^*, e^*) = 0 \text{ implies } \alpha_i^* = 0. \tag{2}$$

This is shown by contradiction. Suppose instead that $\alpha_i^* > 0$ and $\Delta b_i = 0$ (i.e., $e_i^* = e_i^s$). From the incentive trade-off (1), a marginal reduction in α_i will lead only to a second-order increase in Δb_i. But this marginal ownership share can be allocated to a player j such that $e_j^* > e_j^s$, leading to a first-order decrease in Δb_j, thus contradicting the fact that α was efficient.

The theorem follows as a corollary. Part (a) must hold because $\alpha_i^* = 1$ would imply $e_i^* = e_i^s$, and therefore $\Delta b_i = 0$, a contradiction to (2). For part (b), notice that $e_i^* > 0$ together with $\Delta b_i = 0$ would require $e_i^s = e_i^* > 0$, and therefore $\alpha_i > 0$, again a contradiction to (2). ∎

The decreasing incentive trade-off (1) can be used to say more about the efficient value of α. For example, if all players were identical, they would hold equal shares. On the other hand, notice that (1) corresponds to the loss of output when a player reduces effort to her static best response. Thus, if a player is relatively productive, in the sense that a deviation to e_i^s causes a high loss in output, she will receive more shares than her peers.

4. Hidden Efforts

I now turn to the opposite case where efforts are unobservable, and y is sufficiently noisy so that small deviations become the most tempting, as opposed to the global deviations towards e_i^s that were relevant under observable efforts. Section 5 discusses intermediate cases.

Let $y_i \in \mathbb{R}$ be distributed according to the conditional cumulative function $F_i(y_i \mid e_i)$, assumed to be smooth, and decreasing in e_i. This function will not depend on e_{-i}, implying conditional independence across performance signals. Let f_i denote the density of F_i, subscripts will be dropped to simplify notation.

I introduce noise into y through the Mirrlees-Rogerson conditions: assumptions 1 and 2 below (Mirrlees [1979], and Rogerson [1985]). They allow for a first-order approach to incentive compatibility, which means that only local (or infinitesimal) deviations will matter for effort incentives.

Assumption 1: The likelihood ratio $\frac{f_e}{f}(y_i \mid e_i)$ is non-decreasing in y_i for all e_i.
Assumption 2: $F(y_i \mid e_i = c_i^{-1}(z))$ is convex in z for all y_i.

Assumption 1 implies that e_i has a monotonic impact on y_i in terms of first-order stochastic dominance. Assumption 2 means that the marginal impact on y_i of increasing the cost of effort, z, is stochastically decreasing. In other words, the marginal returns to effort in terms of y_i, net of cost, are decreasing. This is a joint condition on F and c_i, and will hold whenever F is never highly concave in e_i so that the convexity of c_i dominates.

As in the previous section, I first derive $\Delta b_i(\alpha_i, e)$, the minimum relational capital required to implement e_i under contract $\langle \alpha, e \rangle$, and then address the dual problem of minimizing the left-hand side of $(IC\text{-}DE)$. It turns out that a cost-effective way to implement e_i is to use a one-step bonus, which takes only two values: $\sup_{y_i} b_i(y_i)$ and $\inf_{y_i} b_i(y_i)$. I first calculate the power required by this bonus, and then show that such power actually constitutes a lower bound.[13]

Suppose contract $\langle \widehat{\alpha}, \widehat{e}\, \rangle$ prescribes an effort $\widehat{e}_i > e_i^s(\widehat{\alpha}_i, \widehat{e}_{-i})$. (The case where $\widehat{e}_i < e_i^s$ will be symmetric, with all signs reversed, and when $\widehat{e}_i = e_i^s$ we simply have $\Delta b_i = 0$.) Consider the problem of deterring local deviations, which are the hardest to detect, and will turn out to be the most tempting in the sense of requiring the highest bonus power among all deviations.[14] Under assumption 1, an infinitesimal deviation to $\widehat{e}_i - \epsilon$ will weakly increase (decrease) the likelihood for all low (high) performance values. In particular, define $\widehat{y}_i$ implicitly by $f_{e_i}(\widehat{y}_i \mid \widehat{e}_i) = 0$, so that all values $y_i < \widehat{y}_i$ have a weakly higher density f under the deviation than under $\widehat{e}_i$, and conversely for $y_i > \widehat{y}_i$. A one-step bonus will exploit this fact by taking the form: $b_i(y_i) = \sup_{y_i} b_i(y_i)$ if $y_i \geq \widehat{y}_i$, and $b_i(y_i) = \inf_{y_i} b_i(y_i)$ otherwise, which will maximally discourage the deviation.

Under one-step bonuses, assumption 2 guarantees that payoffs are concave in effort. These payoffs are now equal to $\alpha_i E\left[x \mid e_i, \widehat{e}_{-i}\right] + \sup_{y_i} b_i(y_i) - \Delta b_i F(\widehat{y}_i \mid e_i) - c_i(e_i)$. Therefore, the incentive constraints (IC) can be replaced by the corresponding first-order conditions. Dropping the tildes on α_i and e_i, these become

$$(FOC) \qquad \alpha_i \frac{\partial}{\partial e_i} E\left[x \mid e\right] - \Delta b_i F_{e_i}(\widehat{y}_i \mid e_i) = c_i'(e_i).$$

The second term $-\Delta b_i F_{e_i}(\widehat{y}_i \mid e_i)$ is equal to $\partial E\left[b_i(y_i) \mid e_i\right] / \partial e_i$, corresponding to the marginal incentives created by the bonus.

[13]This result generalizes the optimality of one-step bonuses derived by Levin [2003] under one-sided moral hazard.

[14]Notice that absent any implicit incentives a player would select e_i^s. But such a deviation (as well as every other deviation) will be deterred as a by-product of deterring local deviations.

From (FOC) we obtain $\Delta b_i = p_i(e_i) - \alpha_i q_i(e)$, where $p_i(e_i)$ and $q_i(e)$ are positive and given by

$$p_i(e_i) := \frac{c_i'(e_i)}{-F_{e_i}\left(\widehat{y}_i \mid e_i\right)}, \text{ and } q_i(e) := \frac{\frac{\partial}{\partial e_i}E\left[x \mid e\right]}{-F_{e_i}\left(\widehat{y}_i \mid e_i\right)}.$$

Recall that the above derivation assumed a prescribed effort $e_i > e_i^s$, and therefore $p_i(e_i) - \alpha_i q_i(e) > 0$. When $e_i < e_i^s$, on the other hand, player i would be tempted to increase effort, and a symmetric reasoning would deliver $\Delta b_i = \alpha_i q_i(e) - p_i(e_i) > 0$. Thus, the general expression for the power required under one-step bonuses is $\left|p_i(e_i) - \alpha_i q_i(e)\right|$.

Proposition 2 shows that $\left|p_i(e_i) - \alpha_i q_i(e)\right|$ imposes a lower bound for the power required by *any* bonus that implements e_i. Constraint $(IC\text{-}DE)$ follows as result:

Proposition 2. *When efforts are not observable, and the Mirrlees-Rogerson conditions hold (assumptions 1 and 2), effort levels e can be implemented under ownership structure α if and only if*

$$(IC\text{-}DE) \qquad \sum_i \Delta b_i(\alpha_i, e) = \sum_i \left|p_i(e_i) - \alpha_i q_i(e)\right| \leq U - \overline{U}.$$

Proof. It remains to show that any bonus $b_i(\cdot)$ that implements e_i, given α_i and e_{-i}, is such that $\Delta b_i \geq \left|p_i(e_i) - \alpha_i q_i(e)\right|$. Assume $e_i > e_i^s$ so that $p_i(e_i) > \alpha_i q_i(e)$ (the opposite case is symmetric), and let $\widehat{y}_i$ be such that $f_{e_i}(\widehat{y}_i \mid e_i) = 0$. Notice that $b_i(\cdot)$ must satisfy, at the very least, the first-order condition for effort selection, which implies

$$\begin{aligned} c_i'(e_i) - \alpha_i \frac{\partial}{\partial e_i} E\left[x \mid e\right] &= \frac{\partial}{\partial e_i} E\left[b_i(y_i) \mid e_i\right] \\ &= \int_{-\infty}^{\widehat{y}_i} b_i(y_i) f_{e_i}(y_i \mid e_i) dy_i + \int_{\widehat{y}_i}^{\infty} b_i(y_i) f_{e_i}(y_i \mid e_i) dy_i \\ &\leq \int_{-\infty}^{\widehat{y}_i} \inf_{y_i} b_i(y_i) f_{e_i}(y_i \mid e_i) dy_i + \int_{\widehat{y}_i}^{\infty} \sup_{y_i} b_i(y_i) f_{e_i}(y_i \mid e_i) dy_i \\ &= -\left[\sup_{y_i} b_i(y_i) - \inf_{y_i} b_i(y_i)\right] \int_{-\infty}^{\widehat{y}_i} f_{e_i}(y_i \mid e_i) dy_i \\ &= -\Delta b_i F_{e_i}(\widehat{y}_i \mid e_i). \end{aligned}$$

The desired inequality follows from dividing both sides by $-F_{e_i}(\widehat{y}_i \mid e_i)$. ∎

I proceed with the dual problem of minimizing the left-hand side of $(IC\text{-}DE)$, given optimal efforts. A simple trick simplifies this problem. Whenever contract

$\langle\alpha, e\rangle$ solves the original problem (II) it is easy to verify that $e_i \geq e_i^s(\alpha_i, e_{-i})$ for all players. The implication is that such inequalities can be added to problem (II) as additional constraints without losing generality. But these constraints are equivalent to $p_i(e_i) - \alpha_i q_i(e) \geq 0$, so once they are added we can drop the absolute values in constraint (IC-DE). As a consequence, the dual problem (for optimal efforts e^*) can be equivalently expressed as

$$\min_{\alpha} \sum_i \{p_i(e_i^*) - \alpha_i q_i(e^*)\} = \max_{\alpha} \sum_i \alpha_i q_i(e^*)$$

subject to:

$$p_i(e_i^*) - \alpha_i q_i(e^*) \geq 0 \text{ for all } i.$$

This is a linear program in α, where the incentive trade-offs are constant and given by $-\partial\Delta b_i/\partial\alpha_i|_{e^*} = q_i(e^*)$. The problem always admits a corner solution where some player $k \in \arg\max_i q_i(e^*)$ receives shares to the point where either $\alpha_k = 1$, or $p_k(e_k^*) - \alpha_k q_k(e^*) = 0$ (i.e., $e_k^s(\alpha_k, e_{-k}^*) = e_k^*$). In both cases we obtain $\Delta b_k = 0$. (Any remaining shares can be allocated in this way to a player with the second highest value of $q_i(e^*)$, and so on, until shares are exhausted.) Whenever $\arg\max_i q_i(e^*)$ is a singleton (a generic property) *any* efficient ownership structure must have this feature. Theorem 2 summarizes the results, while the meaning of $q_i(e^*)$ is addressed below.

Theorem 2. *When efforts are not observable, and the Mirrlees-Rogerson conditions hold (assumptions 1 and 2), there exists an efficient ownership structure such that ownership shares are concentrated in the hands of a single player* $k \in \arg\max_i q_i(e^*)$ *to the point where she receives no implicit incentives* ($\Delta b_k = 0$).

In the generic case where $\arg\max_i q_i(e^*)$ *is a singleton, any efficient ownership structure has the above properties.*

Notice that under first-best efforts we have $e_k^{FB} = e_k^s(1, e_{-k}^{FB})$, and therefore $p_k(e_k^{FB}) = q_k(e^{FB})$. Thus, $\Delta b_k = 0$ implies that ownership must be fully concentrated. Under the second best, however, we can have $e_k^* < e_k^s(1, e_{-k}^*)$, and therefore $p_k(e_k^*) < q_k(e^*)$, implying that Δb_k can become zero before $\alpha_k = 1$.

The stark difference between theorems 1 and 2 arises from the distinct properties of the returns to ownership, which were decreasing under observable actions. The marginal returns to increasing α_i will depend on the size of the shirking temptation that these additional shares mitigate (i.e., when a temptation is large, so is the power of the bonus required to deter it, and therefore additional shares lead to high savings in relational capital). Under observable efforts, returns were decreasing because additional units of α_i deterred ever smaller, and cheaper, global deviations. Under hidden efforts, in contrast, additional shares always mitigate the same local temptation regardless of the size of α_i (which now remains costly to deter due to the difficulty of detecting it), so the resulting marginal savings in relational capital never decrease.[15]

Theorems 1 and 2 give rise to a comparative static with respect to the size of $U - \overline{U}$. When there is no relational capital, and the model becomes a static one, ownership shares will be dispersed among all players exerting positive effort because they become the only source of incentives (and efforts will be low). On the other hand, under unobservable efforts, as relational capital increases it will be used to sustain higher efforts and steeper bonus payments, but the latter not for every player. In fact, the player with the highest trade-off (assuming this player is unique) will receive additional ownership shares, and no bonuses, in order to sustain a higher effort, while other players surrender these additional shares in exchange for steeper implicit incentives. As relational capital approaches the level needed to sustain first-best efforts, a single player receives all shares. When efforts are observable, in contrast, ownership remains dispersed.

A. Who Receives Ownership?

The incentive trade-off $q_i(e^*)$ will be high when a high marginal productivity $\frac{\partial}{\partial e_i} E\left[x \mid e^*\right]$ is combined with a low value for $-F_{e_i}\left(\widehat{y}_i \mid e_i^*\right)$. The latter will be proportional to the statistical power, relative to the size, of an optimal statistical test based on y_i against local deviations $e_i < e_i^*$. This test will have a null hypothesis "$e_i = e_i^*$" and rejection region $\{y_i < \widehat{y}_i\}$, i.e., it is the optimal test of size $F\left(\widehat{y}_i \mid e_i^*\right)$. The statistical power, relative to size, is given by $F\left(\widehat{y}_i \mid e_i\right) - F\left(\widehat{y}_i \mid e_i^*\right)$, and for $e_i = e_i^* - \epsilon$ it is approximately equal to $-F_{e_i}\left(\widehat{y}_i \mid e_i^*\right)\epsilon$. A low value for $-F_{e_i}\left(\widehat{y}_i \mid e_i^*\right)$ therefore corresponds to a poor test, implying that deviations by player i are hard to detect.

Consequently, ownership will be given to a player when she possesses a high marginal productivity combined with a high level of informational asymmetry with respect to her peers, in the sense that her performance is hard to assess. Intuitively, this might be a player that performs a relatively important and complex task.

[15] In Rayo [2002] I also derive extreme ownership results for the case of hidden information regarding the cost of effort. A first-order approach to the resulting revelation problem creates constant returns as well.

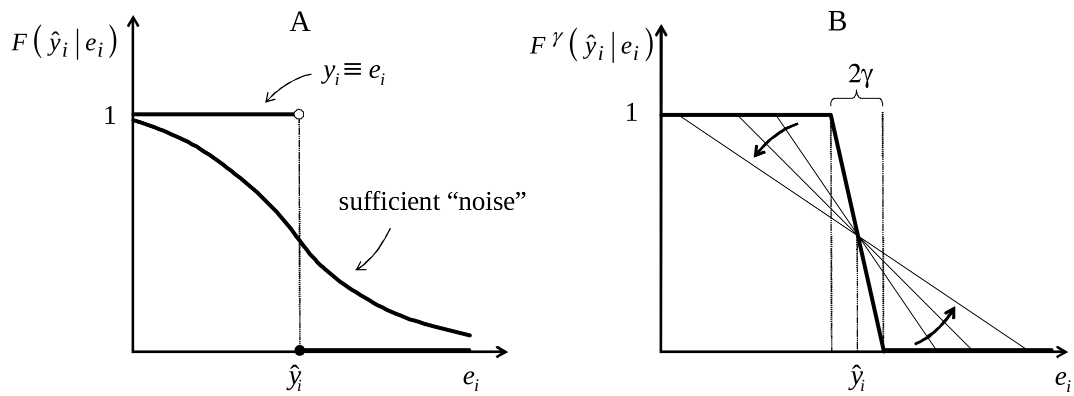

Figure 1. Detection Likelihood

5. Intermediate Cases

The previous sections considered two extremes regarding the information content of y. These extremes can be represented through the likelihood $F(\widehat{y}_i \mid e_i)$ that y_i falls strictly below a given threshold $\widehat{y}_i$, as a function of the effort exerted. This likelihood corresponds to the probability that a player receives a low bonus.[16]

As shown in figure 1.A, when efforts are fully observable, $F(\widehat{y}_i \mid e_i)$ will be equal to 1 whenever $e_i < \widehat{y}_i$, and equal to zero as soon as $e_i \geq \widehat{y}_i$. This function, as well as any continuous approximation of it, will violate assumption 2, which requires that the combination of $F(\widehat{y}_i \mid e_i)$ with the convex cost function $c_i(\cdot)$ be convex. But observable (or nearly observable) efforts will imply that $F(\widehat{y}_i \mid e_i)$ is extremely concave to the left of $\widehat{y}_i$, and this curvature will dominate. Conversely, a function $F(\widehat{y}_i \mid e_i)$ satisfying assumption 2 will be one that is never too concave so that the convexity of $c_i(\cdot)$ now dominates.

The purpose of this section is to extend the ownership results to intermediate cases where neither of these two extremes hold, and therefore both local and global (i.e., non-infinitesimal) deviations play a role. The plan is to parameterize $F(\widehat{y}_i \mid e_i)$ with a measure of noise, $\gamma \geq 0$, which creates a continuous transition, as γ increases, from the case of fully observable efforts where ownership is dispersed to the case where only local effort constraints bind and ownership is concentrated. This comparative static will be performed for the dual minimization problem under fixed efforts e^*.[17] For

[16]Defining $F(\widehat{y}_i \mid e_i) := prob\{y < \widehat{y}_i \mid e_i\}$, instead of the customary $prob\{y \leq \widehat{y}_i \mid e_i\}$, allows for this useful connection.

[17]A comparative static for the original problem would need to consider the impact of γ over e^*, but the same type of ownership transition will occur.

concreteness I assume $e^* = e^{FB}$, but theorem 2 will suggest how the results extend to second-best efforts (where ownership is never concentrated beyond the point of zero implicit incentives).

Intuitively, what is required is that, as γ increases, local deviations become ever harder to detect relative to global deviations. Since the bonus power needed to deter a given deviation is inversely related to the ease of detecting such a deviation, as $\gamma \to \infty$ the bonus power required to deter local deviations will eventually exceed that required for each global deviation. Once this occurs, returns to ownership become constant, and ownership is concentrated.

I first work out a simple example with uniform distributions, and then show how the results extend to other familiar distributions. In particular, let y_i be uniformly distributed between $e_i - \gamma$ and $e_i + \gamma$. Figure 1.B shows how the parameterized likelihood, $F^\gamma(\widehat{y}_i \mid e_i)$, rotates with γ for a fixed cutoff $\widehat{y}_i$.

For this example, one can show that it is optimal to use one-step bonuses with a cutoff equal to $\widehat{y}_i = e_i^* - \gamma$. The reason this cutoff is optimal is that it maximizes, for any given deviation to e_i, the size of $F^\gamma(\widehat{y}_i \mid e_i) - F^\gamma(\widehat{y}_i \mid e_i^*)$, namely, the increase in the probability that a low bonus is paid upon such a deviation. Notice that the probability of a low bonus is zero when e_i^* is selected. This probability increases linearly in the size of the deviation for deviations within $(e_i^* - 2\gamma, e_i^*)$, and for deviations to efforts lower than $e_i^* - 2\gamma$ the low bonus is paid with certainty. Finally, as γ increases, the detection likelihood $F^\gamma(e_i^* - \gamma \mid e_i)$ will pivot around e_i^*.

This example is convenient because the bonus power takes a simple form:

$$\Delta b_i(\alpha_i, e^*) = \max\{\varphi_i(\alpha_i, e^*), p_i(e_i^*) - \alpha_i q_i(e^*)\},$$

where the functions on the right-hand side are those derived in sections 3 and 4. To see that the right-hand side is *sufficient* to deter all potential deviations, we can separate these into the intervals $[0, e_i^* - 2\gamma]$ and $(e_i^* - 2\gamma, e_i^*)$. Any deviation to an effort in $[0, e_i^* - 2\gamma]$ will be detected with probability one. As a result, from section 3, a bonus power equal to $\varphi_i(\alpha_i, e^*)$ will be sufficient to deter every deviation in this range. Over the second interval, on the other hand, assumption 2 will be satisfied (since $F^\gamma(\widehat{y}_i \mid e_i)$ is linear in e_i), and therefore, from section 4, a bonus power equal to $p_i(e_i^*) - \alpha_i q_i(e^*)$ will suffice for this range. To see that the right-hand side is *necessary* to implement the desired effort, notice that deterring a deviation to e_i^s requires a bonus power of at least $\varphi_i(\alpha_i, e^*)$ (it will require an even larger power when e_i^s cannot be detected with certainty). Moreover, deterring local deviations requires at least $p_i(e_i^*) - \alpha_i q_i(e^*)$.

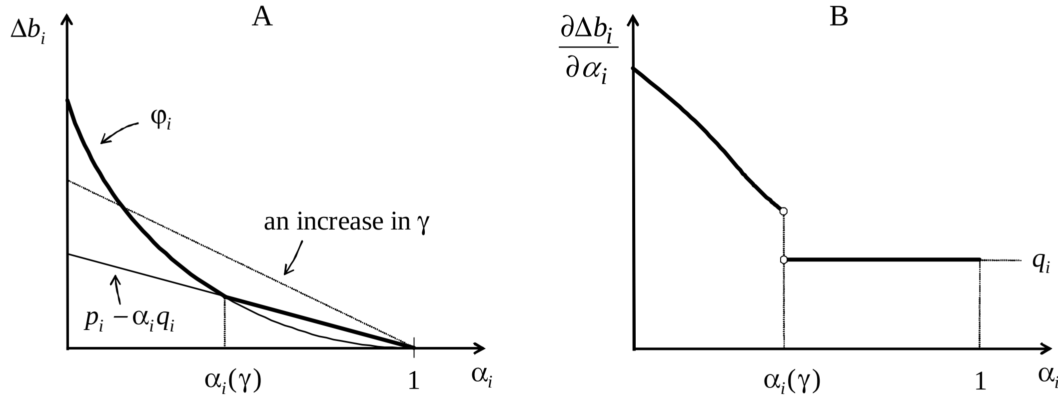

FIGURE 2. Decreasing/constant returns

Which one of the functions $\varphi_i(\alpha_i, e^*)$ and $p_i(e_i^*) - \alpha_i q_i(e^*)$ is larger will depend on α_i in a simple way. As illustrated in figure 2.A, both functions are decreasing in α_i, and equal to zero when $\alpha_i = 1$. (Recall also that $\partial\varphi_i/\partial\alpha_i = 0$ when $\alpha_i = 1$.) Due to the convexity of $\varphi_i(\alpha_i, e^*)$, there exists a unique level of ownership $\alpha_i(\gamma) \in [0, 1]$ satisfying: $\varphi_i(\alpha_i, e^*) > p_i(e_i^*) - \alpha_i q_i(e^*) \Leftrightarrow \alpha_i < \alpha_i(\gamma)$. The solution for the dual problem $\min_\alpha \sum_i \Delta b_i(\alpha_i, e^*)$ will follow from the properties in remark 1, illustrated in figure 2.B:

Remark 1. *In the model where y_i is distributed uniform $[e_i \pm \gamma]$ we have:*[18]

(a) $-\partial\Delta b_i/\partial\alpha_i|_{e^*} = \begin{cases} q_i(e^*) & \text{if } \alpha_i > \alpha_i(\gamma), \\ -\frac{\partial}{\partial\alpha_i}\varphi_i(\alpha_i, e^*) > q_i(e^*) & \text{if } \alpha_i < \alpha_i(\gamma). \end{cases}$

(b) *$\alpha_i(\gamma)$ is decreasing in γ, with $\alpha_i(0) = 1$, and $\alpha_j(\gamma) \to 0$ as $\gamma \to \infty$.*

To see how $\alpha_i(\gamma)$ depends on γ, consider an increase in the latter. Its impact will be through the marginal statistical power $-F_{e_i}^\gamma(\widehat{y}_i \mid e_i)$, now equal to $1/(2\gamma)$. A lower statistical power leads to an increase in the bonus power required for local deviations (graphically, $p_i(e_i^*) - \alpha_i q_i(e^*)$ will pivot upward). The function $\varphi_i(\alpha_i, e^*)$, in turn, will not be affected by the higher γ. As a consequence, $\alpha_i(\gamma)$ will decrease.

Notice, finally, that $-\partial\Delta b_i/\partial\alpha_i|_{e^*}$ is bounded below by $q_i(e^*)$ (because even marginal deviations are expensive to deter using bonuses). The implication is that no player j outside the set $\arg\max_i q_i(e^*)$ (generically a singleton) will receive ownership beyond $\alpha_j(\gamma)$, as these extra shares are best allocated among the players inside $\arg\max_i q_i(e^*)$.

[18]When $\alpha_i = \alpha_i(\gamma)$ only directional derivatives will exist.

The results are as follows. When γ is small, $\alpha_j(\gamma)$ will be close to one, and the dual problem will be dominated by decreasing returns that lead to dispersed ownership. On the other hand, as γ increases, $\alpha_j(\gamma)$ will decrease, and all players j outside $\arg\max_i q_i(e^*)$ will surrender ownership in favor of players within this set so that $\alpha_j \leq \alpha_j(\gamma)$. In the extreme, since $\alpha_j(\gamma) \to 0$ as $\gamma \to \infty$, all shares are eventually transferred to the players, or a single player, in $\arg\max_i q_i(e^*)$.

A. Other Distributions

Consider a more general approach. Let $y_i = e_i + z$, where z is a random variable distributed according to a density $g^\gamma(z)$ that is non-decreasing in z over $\mathbb{R}_-$, non-increasing over $\mathbb{R}_+$, and smooth around $z = 0$ (except for a possible kink at $z = 0$). Let γ parameterize the dispersion of z. Suppose that an increase in γ "flattens" $g^\gamma(\cdot)$ so that, for all $z \neq 0$: $g^\gamma(z)/g^\gamma(0) \to 1$ and $g^\gamma_{z+}(0)/g^\gamma(0) \to 0$ as $\gamma \to \infty$, and $g^\gamma(z) \to 0$ as $\gamma \to 0$. Some common examples satisfying these conditions include the normal, double-exponential, triangular, and the uniform used above. For each of these distributions the parameterization of dispersion is unambiguous.

The density of y_i conditional on e_i is given by $f^\gamma(y_i \mid e_i) := g^\gamma(y_i - e_i)$. Monotonicity of $f^\gamma(y_i \mid e_i)$ in y_i, on either side of e_i, implies that one-step bonuses remain optimal for local deviations (thus replacing assumption 1), and their threshold $\widehat{y}_i$ can be optimally set equal to e_i^*. I again focus for concreteness on the case where $e^* = e^{FB}$.

The above results for the uniform extend as follows: *For each player there exists an ownership threshold $\alpha_i(\gamma)$ such that $-\partial \Delta b_i / \partial \alpha_i|_{e^*} = q_i(e^*)$ for all $\alpha_i > \alpha_i(\gamma)$, and such that $\alpha_i(\gamma) \to 0$ as $\gamma \to \infty$.* This again implies that all players outside $\arg\max_i q_i(e^*)$ will surrender all their shares to those within this set as $\gamma \to \infty$. As $\gamma \to 0$, on the other hand, mitigating small deviations becomes arbitrarily cheap, so global constraints again bind and ownership is dispersed.

The argument is the following. From (IC), the power required to deter any global deviation to e_i, using a one-step bonus with cutoff e_i^*, is given by

$$\frac{c_i(e_i^*) - c_i(e_i) - \alpha_i \left\{ E[x \mid e^*] - E[x \mid e_i, e_{-i}^*] \right\}}{F^\gamma(e_i^* \mid e_i) - F^\gamma(e_i^* \mid e_i^*)}. \tag{3}$$

Notice that $p_i(e_i^*) - \alpha_i q_i(e^*)$ is the limit of (3) as $e_i \to e_i^*$. This local power will be larger than (3), and therefore suffice to deter a global deviation to e_i, whenever

$$\frac{c_i(e_i^*) - c_i(e_i) - \alpha_i \left\{ E[x \mid e^*] - E[x \mid e_i, e_{-i}^*] \right\}}{(e_i^* - e_i) \left\{ c'(e_i^*) - \alpha_i \frac{\partial}{\partial e_i} E[x \mid e^*] \right\}} \leq \frac{F^\gamma(e_i^* \mid e_i^*) - F^\gamma(e_i^* \mid e_i)}{(e_i^* - e_i)\, F^\gamma_{e_i}(e_i^* \mid e_i^*)}. \tag{4}$$

The left-hand side is smaller than 1 (due to strict convexity of c_i and concavity of $E[x \mid e]$), does not depend on γ, and will decrease towards a negative value as α_i increases. The right-hand side is positive, and will converge to 1 as $\gamma \to \infty$ (since $f^\gamma(e_i^* \mid e_i)/f^\gamma(e_i^* \mid e_i^*) \to 1$). This is precisely the condition that, as γ increases, local deviations become harder to detect relative to global deviations.

Now fix γ and $e_i < e_i^*$, and consider the smallest value of α_i such that inequality (4) holds. Call this value $\alpha_i(\gamma, e_i)$. It is continuous in e_i and converges to zero as $\gamma \to \infty$. Also define $\alpha_i(\gamma) := \sup_{e_i < e_i^*} \alpha_i(\gamma, e_i)$, so that (4) holds for all e_i whenever $\alpha_i > \alpha_i(\gamma)$. Only local constraints bind beyond this point and therefore $-\partial \Delta b_i / \partial \alpha_i |_{e^*} = q_i(e^*)$ for all $\alpha_i > \alpha_i(\gamma)$.[19]

It remains to show that $\alpha_i(\gamma) \to 0$ as $\gamma \to \infty$, which is equivalent to $\alpha_i(\gamma, e_i) \to 0$ uniformly in $e_i \in [0, e_i^*)$. This may fail due to the openness on the right. However, a technical point to note is that there exists a neighborhood $(e_i^* - \epsilon, e_i^*]$ of positive length such that: for all large γ, $\alpha_i(\gamma, e_i) = 0$ for all $e_i \in (e_i^* - \epsilon, e_i^*]$. (This follows from strict convexity of $c(\cdot)$, and the fact that g^γ flattens out with γ so that $g^\gamma_{z+}(0)/g^\gamma(0) \to 0$.)[20] The implication is that uniform convergence of $\alpha_i(\gamma, e_i)$ need only hold over the closed interval $[0, e_i^* - \epsilon]$, which is guaranteed by the continuity of $\alpha_i(\gamma, e_i)$ in e_i.

6. Discussion

The analysis thus far has focused on the allocation of ownership shares and bonus incentives. I now turn to the budget-balancing task of *paying* the performance bonuses, for which the model allows considerable liberty. In fact, this task can be given to any member as long as she is also allocated sufficient relational capital (in the form of continuation surplus) so that she honors her promises. Fully transferrable relational capital will then imply that any player is equally suited, and the details of the arrangement are immaterial. In practice, however, asymmetries are likely to arise. If some players, for instance, have comparative advantages to holding future surplus, they will also have advantages to paying bonuses.[21] Players might also face

[19] The range of constant returns might extend below $\alpha_i(\gamma)$, since setting a bonus threshold lower than e_i^* may be useful when dealing with global deviations (as in the uniform case above).

[20] Using a Taylor expansion to the left of $e_i = e_i^*$ for both $c_i(e_i)$ and $F^\gamma(e_i^* \mid e_i)$, and rearranging terms, when $\alpha_i = 0$ inequality (4) is equivalent to $c''(e_i^*)/c'(e_i^*) \geq -g^\gamma_{z+}(0)/g^\gamma(0) + o(e_i^* - e_i)$. (Notice that $F^\gamma_{e_i}(e_i^* \mid e_i) = -g^\gamma(e_i^* - e_i)$). But $c''(e_i^*)/c'(e_i^*)$ is larger than (and remains bounded away from) $-g^\gamma_{z+}(0)/g^\gamma(0)$ for all large γ, so the inequality will hold for all e_i close to e_i^*.

[21] For example, MacLeod and Malcomson [1998] develop a model where the form of the labor market determines how future surplus must be allocated. In addition, players might have different horizons or intertemporal rates of substitution.

liquidity constraints that interfere with high payments, or they may have differential knowledge about the performance measures of their peers.

When efforts are not observable and the (main) residual claimant of output receives no additional incentives, there will be some potential advantages to charging her with the payment task as well. Since this residual claimant need not be compensated by her peers, such arrangement minimizes the quantity of information required to actually carry out the relational contract. The only requirement is that she observes the productivity of her peers, while they need only be aware of their own productivity measures in order to verify that the contract is honored. (In the other extreme, when the payment task is fully dispersed, every player requires information concerning all of her peers, adding up to an order-of-magnitude more information.) In addition, since the residual claimant receives the profits of the firm, she is likely to have liquidity, again favoring the arrangement.

On the other hand, when efforts are observable and ownership is dispersed, it is always possible to cast implicit incentives in terms of punishments that arise only off the path of play, instead of prizes following high effort. As a consequence these incentives need not take the form of systematic money transfers across partners.

In terms of potential applications, the model would best resemble closely-held firms. Within this class, shared ownership is dominant in professional services (e.g., law, accounting, investment banking, management consulting, advertising, architecture, engineering, and medicine) where, consistent with the model, “the quantity and quality of each individual’s inputs and outputs can be observed with relative ease” and “the productivity of individual employees can be, and generally is, monitored remarkably closely”, Hansmann [1996, p. 70]. Such firms are typically composed of a relatively small number of individuals of the same profession, who understand well the tasks of their peers. Their work is also relatively independent from each other and thus easier to assess.

Employee ownership, on the other hand, rarely occurs in the industrial sector or in non-professional services (e.g., hotels and retailing), where a larger task and background diversity is likely to entail informational asymmetries.[22] Centralized ownership and monitoring are the trade-mark of what Alchian and Demsetz call the “classical firm”. Underlying this widespread form of organization, they observe, is a difficulty to detect performance. They in fact view clustered ownership precisely

[22] For example, it will tend to be easier for a lawyer to assess the performance of one of her partners, than for an industrial worker to assess the performance of her CEO.

as a response to the problem of asymmetric information, i.e., the owner monitors employees, and her incentives emerge from the residual claim of earnings.[23] I also identify informational asymmetry with concentrated ownership, but through a different channel: the optimal use of relational capital.

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