MIT 14.380
Probability Self-Study Module
Summer 2006

Course Website (contains all relevant materials): `http://web.mit.edu/14.380/www`

Developer: Raymond P. Guiteras

Time and Location: None.

Office Hours: None

Primary Textbooks:
Morris H. DeGroot and Mark J. Schervish. *Probability and statistics*. Addison Wesley, 3rd edition, 2002. URL `http://lib.stat.cmu.edu/~mark/degroot/.index.html`
George Casella and Roger L. Berger. *Statistical Inference*. Duxbury, 2nd edition, 2002. URL `http://www.stat.ufl.edu/~casella/class/errata1.pdf` (This will be a required text for 14.381 in the fall.)

Supplementary Textbooks:
Sheldon M. Ross. *A First Course in Probability*. Prentice-Hall, 5th edition, 1998 (An undergrad textbook with very clear exposition and lots of good examples and problems)
Geoffrey R. Grimmett and David R. Stirzaker. *Probability and Random Processes*. Oxford University Press, 3rd edition, 2001 (A masters / grad level book. For those who want to be econometricians.)
Sheldon M. Ross. *An Introduction to Probability and Statistics for Engineers and Scientists*. Academic Press, 3rd edition, 2004 (Similar to Ross (1998) above, but contains a nice introduction to statistics (estimation and inference) that you may find useful in the fall.)

Overview: This self-study course in probability theory is designed to prepare you for 14.381 in the fall. It consists of readings from DeGroot and Schervish (2002), Chapters 1-5, and Casella and Berger (2002), Chapters 1-5 and of practice exercises from undergraduate and graduate courses previously offered in the MIT Economics department.

Doing the practice exercises is the most important part of the course – if this material is new to you, you will not learn it well just from reading the text. If you have already taken probability and are not sure if you need to work through this course, try a stratified random sample of the practice exercises as a self-test. (If you don't know what a "stratified random sample" is, then you need to do this course.)

The problems are drawn from MIT 14.30, Fall 2005 and MIT 14.381, Fall 2005 (previous editions of 14.381 included a unit on probability), and problem sets will be labelled as such. There may be some overlap in these problems; please let me know of any redundancies. Problems may refer to "the text" or "the book": for 14.30, the text was DS; for 14.381, it was CB.

Those of you who have a strong background in probability (e.g. masters-level coursework) and need only a review may choose to concentrate on the CB readings and 14.382 problems. Those of you with less strong probability backgrounds (e.g. just an introductory undergraduate probability course) should do the readings from both DS and CB and the problems from 14.30 and 14.382.

Grading: None.

Exams: None.

Outline of the Course:

1. *Foundations and Counting Methods*

   Readings: DS 1.4-1.10; CB 1.1-1.2

   Do not spend too much time or mental effort on the subtleties of the different counting methods. You should be familiar with the basic formulae and when to apply them.

   Problems: *14.30:* DS 1.4-1.10, 1.12 odd-numbered exercises (answers are in the back of the book); PSet 1, Q 1 & 2.; *14.381:* PSet 1, Q 1-3.

2. *Conditional probability, Independence and Bayes' Rule*

   Readings: DS 2.1-2.3; CB 1.3

   DS 2.3 covers the Law of Total Probability and Bayes' Theorem, both of which are *crucial* in statistics and economics. Be sure to study this section especially carefully and do as many of the section-end exercises as you can. DS 2.4, on Markov Chains, is not necessary for the econometrics sequence, although the concept does show up in many areas of economics and econometrics.

   Problems: *14.30:* Practice Problems Lecture 4 & 5; PSet 1 Q 3-5, PSet 2; Exam 1 Q 1&5; *14.381:* PSet 1 Q 5-7

3. *Random variables and their distributions*

   Readings: DS 3.1-3.3; CB 1.4-1.6

   DS 3.1-3.2 introduces discrete and continuous random variables but does not give many examples along the way. I think it is easier to understand the concepts with a couple of examples in mind, so I recommend looking at *randomvariableexamples.pdf*, which contains some simple examples we used in 14.30. Some 14.30 students struggled with the material on quantiles in Section 3.3 and found the handouts *quantiles.pdf* and *quantiles_ supplement.pdf* helpful (the latter refers to the standard normal distribution, which is not covered until DS 5).

   Problems: *14.30:* Practice problems: Lecture 6, Lecture 7 Q 1-4; PSet 3 *14.382:* PSet 1 Q4; PSet 2 Q 1-3

4. *Bivariate and multivariate distributions; marginal and conditional distributions*

   Readings: DS 3.4-3.7; CB 4.1-4.2, 4.6

   I find the notation DS use for marginal, joint and conditional distributions and transformation to be somewhat confusing, so I follow a slightly different notation: see *notationhandout.pdf*. For more on the multivariate versions of the Law of Total Probability and Bayes' Rule, see *lec9lotpbr.pdf*.

   Problems: *14.30:* Practice problems: Lecture 7 Q 5; Lecture 8 (all); Lecture 9 (all); Exam 1 Q 2, 3, 4, 6 (note that 2a is not a very well-posed question)

5. *Transformations of random variables and random vectors*

   Readings: DS 3.8-3.9; CB 2.1, 4.3

   See *lec11supplement.pdf* and *lec12supplement.pdf* for further exposition of the material from Sections 3.8-9 on transformations of random variables and random vectors. The last of these handouts requires some familiarity with matrix algebra (determinants).

   Problems: *14.30:* Practice problems: Lecture 10 (all), Lecture 11 (all), Lecture 12 Q 2-3 (you should be able to do the background (sum of two uniforms has the "triangle" density) to Q2); PSet 4 (note: PSet 4 Q 2 is actually a special case of a more general result discussed in DS page 161 ("The Probability Integral Transformation"), which is very useful for the simulation exercses you will do in the second half of 14.381); Exam 1 Q 6; Exam 2 Q 1, 2, 6; *14.382:* PSet 2 Q 2-3

6. *Expectation, Moments and Correlation*

   Readings: DS 4 (all sections); CB 2.2-2.3, 4.3-4.5, 4.7, 5.1-5.3.1

   Moment generating functions (described in DS 4.4 and CB 2.3) are useful to econometricians and for some theoretical results (in particular, showing relationships among certain random variables and proving the central limit theorem), but are not necessary in applied work or for the econometrics sequence, so you may skim the material under "Moment Generating Functions" and "Properties of Moment Generating Functions" (DS. 205-209). However, you should read the section "Existence of Moments" (DS. 203-205) carefully.

   The material in DS 4.5 and 4.7 on optimal predictors under mean squared error loss are particularly important for the econometrics sequence. However, some of the students found the exposition in DS confusing. You may find it helpful to read the handout *DS4point7.pdf* before reading the "Prediction" section in DS 4.7.

DS 4.9 on utility and statistical decision theory is not strictly necessary for econometrics, so you may skim it in this course, but is a lot of fun and will help you in micro (and macro, and labor, and PF, and...).

Problems: *14.30:* Practice problems: Lecture 12 Q 1, Lectures 13-16 (all); PSet 5; Exam 2 Q 3, 4, 5, 7.; *14.382:* PSet 3 Q 1-4; PSet 4 Q 2; *Note:* Conditional expectations and the Law of Iterated Expectations (DS call this the "Law of Total Probability for Expectations") come up again and again in econometrics. To test your understanding of these concepts, work your way through Section 2.2.4 and Appendix 2.A.1 of Wooldridge (2001) and see if you can prove the following results: (1) $E\left[Yg\left(X\right)|X\right] = E\left[Y|X\right]g\left(X\right)$ (this is known as the "pull-through property" or "linearity of conditional expectations"); (2) $E\left\{E\left[Y|X,Z\right]|X\right\} = E\left[Y|X\right] = E\left\{E\left[Y|X\right]|X,Z\right\}$ (this is known as the "tower property" or sometimes as the "law of iterated expectations")

7. *Special Distributions and the Central Limit Theorem*

   Readings: DS, 5 (5.8 is optional), 7.2, 7.4, 8.7; CB 3, 5.3.2

   Most of the key results are summarized in the handouts *commondistributions.pdf*. For each random variable, you should know the "key things" referenced on page 1 of the handout, with the exception of the MGF and MLE. DS introduce the t-, F- and chi-square distributions in the context of estimation (in chapters 7 and 8), but for an introduction you may see *tdistrib.pdf*, *chisquared.pdf* and *fdistrib.pdf*.

   Problems: Lecture 17-19 practice problems; PSet 6 (note: Q1 refers to *hypergeometric.pdf*); Exam 3 Q 5 *14.382:* PSet 2 Q 4, 5, 6; PSet 3 Q 5; PSet 4 Q 3-4

8. *Review*

   To test your understanding, do the Practice Midterm and the Midterm for 14.381.

References

George Casella and Roger L. Berger. *Statistical Inference*. Duxbury, 2nd edition, 2002. URL `http://www.stat.ufl.edu/~casella/class/errata1.pdf`.

Morris H. DeGroot and Mark J. Schervish. *Probability and statistics*. Addison Wesley, 3rd edition, 2002. URL `http://lib.stat.cmu.edu/~mark/degroot/.index.html`.

Geoffrey R. Grimmett and David R. Stirzaker. *Probability and Random Processes*. Oxford University Press, 3rd edition, 2001.

Sheldon M. Ross. *A First Course in Probability*. Prentice-Hall, 5th edition, 1998.

Sheldon M. Ross. *An Introduction to Probability and Statistics for Engineers and Scientists*. Academic Press, 3rd edition, 2004.

Jeffrey M. Wooldridge. *Econometric Analysis of Cross Section and Panel Data*. MIT Press, 2001. URL `http://www.msu.edu/~ec/faculty/wooldridge/book2.htm`.