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Midterm Examination

Instructions: This is a closed book exam. You have 90 minutes to answer the questions. Good luck!

1. Let A and B be events with positive probability. Show the following:

   a) If A and B are independent then A and B^c are independent.

   b) If $P(A|B) < P(A)$ then $P(B|A) < P(B)$.

2. Prove or disprove the following: If $E(Y|X) = X$ and $E(X|Y) = Y$ and both EX^2 and EY^2 are finite then $P[X = Y] = 1$ (Hint: Show that $\text{Var}(X - Y) = 0$).

3. Let $X = [X_1, X_2]'$ have a bivariate normal distribution with parameters $\mu = 0$ and

$$\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{bmatrix}.$$

   a) Write down the marginal density of X_1 and the conditional density of X_1 given X_2. Find the first two (conditional) moments of the conditional distribution (Hint: you can use known properties of the normal distribution without proof).

   b) Let $\varepsilon(b) = X_1 - bX_2$. Find the value for the parameter b, expressed in terms of the parameters Σ, such that $E[\varepsilon(b)]^2$ is minimal.

4. Let X be a $n \times 1$ vector of random variables with $EX = \mu$ and $E(X - \mu)(X - \mu)' = \Sigma$. Note that X is not assumed to be multivariate normal.

   a) Let A be a $n \times n$ matrix and b an $n \times 1$ vector. Find the mean vector and covariance matrix of $Y = AX + b$.

   b) Show that $EX'AX = \text{tr}(A(\Sigma + \mu\mu'))$.

5. Let X and Y be random variables with joint density

$$f_{X,Y}(x, y) = \begin{cases} e^{-(x+y)} & x > 0, y > 0 \\ 0 & \text{otherwise} \end{cases}.$$

   Let $Z = X + Y$.

   a) Find the joint density of Z and X (Hint: be careful to state where the density is defined).

   b) Find the marginal density of Z and the marginal density of X.

   c) Find the conditional density of Z given $X = x$.