14.381 Problem Set 3
Statistics Fall, 2004

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Due October 4 (6:00pm), in E52-204.

1. (a) State the pdf of the gamma(α, β) distribution.
   (b) Using the fact that the pdf integrates to one, derive the mean and variance of the gamma distribution.
   (c) Derive the mgf of the gamma distribution.
2. C&B 3.8
3. C&B 3.9
4. C&B 3.20
5. C&B 3.23
6. Let X be a random variable with PDF $f(x)$. The density of a truncated distribution is given by: $f(x|x > a) = \frac{f(x)}{\Pr(x>a)}$
   (a) $X \sim U(0,1)$. Find the PDF for the truncated random variable:$f(x|x > a)$ for any $a \in (0,1)$
   (b) Determine the mean and the variance of that truncated random variable
   (c) $X \sim N(\mu, \sigma^2)$. Find the PDF for the truncated random variable: $f(x|x > a)$
   (d) Verify that the mean and the variance of the truncated distribution are

$$\begin{aligned} E(X|X > a) &= \mu + \sigma\lambda(\alpha) \\ Var(X|X > a) &= \sigma^2\{1 - \lambda(\alpha)[\lambda(\alpha) - \alpha]\} \end{aligned}$$

   Where $\phi(x)$ and $\Phi(x)$ are the PDF and CDF of the standard normal distribution $N(0,1)$, $\alpha = \frac{a-\mu}{\sigma}$ and $\lambda(\alpha) = \frac{\phi(\alpha)}{1-\Phi(\alpha)}$