Question 2 part (c)
Maintain the assumption that $\theta_1 = 0$. Then

$$Cov\left(\overline{X}, S^2\right) = \frac{1}{2n^2(n-1)} E\left\{\sum_{k=1}^{n} X_k \sum_{i=1}^{n}\sum_{j=1}^{n} (X_i - X_j)^2\right\}$$

The double sum over i and j has $n(n-1)$ nonzero terms. For each of these, the entire expectation is nonzero for only two values of k (when k matches either i or j). Thus

$$Cov\left(\overline{X}, S^2\right) = \frac{2n(n-1)}{2n^2(n-1)} E\left[X_i (X_i - X_j)^2\right] = \frac{1}{n}\theta_3,$$

and $\overline{X}$ and S^2 are uncorrelated if $\theta_3 = 0$.